

(Unoriented) T-folds with few T's

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(Unoriented) T-folds with few T's

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ABSTRACT: We use the combined action of \mathbb{Z}_2 -chiral reflections (T-dualities) and shifts to build $\mathcal{N} = 1, 2$ supersymmetric four-dimensional string compactifications with few moduli. In particular, we consider \mathbb{Z}_2^4 asymmetric orbifolds of Type IIB on the maximal torus of $SO(12)$ that mimic $\mathcal{N} = 2$ Calabi-Yau compactifications with small “effective” Hodge numbers starting from $(h_{11}, h_{21}) = (1, 1)$. We analyze possible unoriented projections, providing Type I examples with or without open strings. $\mathcal{N} = 1$ oriented asymmetric shift-orbifolds of Type IIB with few chiral multiplets are also presented.

KEYWORDS: Flux compactifications, D-branes, Superstring Vacua, String Duality

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1 Introduction and summary

Moduli stabilization in string theory is a long-standing crucial issue if one is to make contact with low energy (accelerator) physics [1]. A very promising path is to turn on fluxes along the internal directions that generate a potential for the moduli fields [2]. However quantization of strings in the presence of geometric fluxes is only possible for very special choices involving only open string fluxes [3]. Many interesting cases, such as combinations of NS-NS and R-R closed string fluxes, are only amenable to an analysis in the low-energy supergravity approximation. At the same time, a ten-dimensional perspective has a hard time describing non-geometric fluxes and torsions that may admit perfectly consistent description in four dimensions as effective (gauged) supergravities [2]. Quite remarkably, resorting to non-geometric constructions, yet based on exactly solvable (rational) CFT, it is possible to stabilize many if not all the closed string moduli at tree level or in perturbation theory. One can then turn on allowed fluxes or invoke perturbative and non perturbative effects (such as D-brane instantons) to stabilize the remaining moduli.

Aim of this paper is to further explore controllable mechanisms (in string perturbation theory) of moduli stabilization based on exactly solvable (rational) CFT's. We will present simple non-geometric examples of asymmetric orbifolds [6] of special tori or, equivalently, free fermionic constructions [4, 5] with few moduli. The strategy we adopt rests on the simple observation that chiral (and thus non-geometric) twists tend to freeze out untwisted moduli while shifts tend to eliminate twisted ones [7]. Asymmetric orbifolds of Type IIB involving chiral twists with no shifts have been previously studied in [8].

The simplest non-geometric twist one can think of, is a $\mathbb{Z}_{2L,R}$ chiral reflection acting on the Left or Right moving closed string modes. This is nothing but an element of the T-duality group acting on the worldsheet fields. Here we combine T-duality twists of this type with asymmetric shifts to build $\mathcal{N} = 2$ compactifications of Type IIB with few moduli. More precisely, we consider $\mathbb{Z}_{2L}\sigma_A \times \mathbb{Z}'_{2L}\sigma_B \times \mathbb{Z}_{2R}\bar{\sigma}_C \times \mathbb{Z}'_{2R}\bar{\sigma}_D$ orbifolds of Type IIB on the maximal T^6 torus of $SO(12)$ with σ 's some half-shifts. We obtain several models with low “effective” Hodge numbers starting from $(h_{11}, h_{21}) = (1, 1)$. The construction admits a simple description in terms of free fermions that allows a systematic search by computer means.

In view of the possibility of performing an unoriented projection and including D-branes and open strings, we mainly focus on non-geometric Type IIB models in four dimensions with chiral actions on Left-movers mirrored by identical actions on the Right-movers [9, 10]. $\mathcal{N} = 1$ vacua, following from non-geometric orbifolds of Type IIB involving $(-)^{FR}$ projections breaking all the susy from the Right-movers will be also considered.¹ In both cases we find $\mathcal{N} = 1$ models with vector multiplets and few chiral multiplets.

Some comments on the subtle role played by discrete moduli in asymmetric orbifolds are in order. Asymmetric orbifolds typically require specific choices of the internal lattice where “untwisted” moduli (metric and B-field) are frozen to specific values. As one is exploring different branches of the original moduli space, even geometric projections give rise to peculiar twisted spectra [12]. To be specific, starting with the maximal torus of $SO(12)$ the number of twisted sectors gets reduced from 48 (16 per each twist) to 12 with a different chirality structure. As a result, a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold of the $SO(12)$ torus has “effective” Hodge numbers $(h_{11}, h_{21}) = (15, 15)$ rather than $(h_{11}, h_{21}) = (51, 3)$ or $(h_{11}, h_{21}) = (3, 51)$ as expected when the off-diagonal components of G and B are set to zero [13]. The somewhat analogous peculiarities resulting from turning on a discrete quantized value for the B-field, originally observed in [14] and then in [15–17], has been recently reanalyzed in [18, 19].

The plan of the paper is as follows. In section 2 we sketch the idea of perturbative moduli stabilization by means of (T-duality) twists and shifts. In section 3 we describe the basic ingredients of the free fermionic construction with particular attention to the case of chiral \mathbb{Z}_2 actions. In section 4 we present the results of a systematic search over consistent \mathbb{Z}_2^4 orbifolds of Type IIB models with $\mathcal{N} = 2$ susy that admit natural projections to unoriented $\mathcal{N} = 1$ theories. In particular, we analyze in some details the “minimal” model with “effective” Hodge numbers $(h_{11}, h_{21}) = (1, 1)$, that seems to have escaped previous scans in the literature [20, 21]. In section 5 we describe oriented Type II models with $\mathcal{N} = 1$ susy. In section 6, we present an unoriented model without D-branes based on the

¹Being non Left-Right symmetric, these models do not admit a natural unoriented projection but can be coupled to generalized D-branes of the kind proposed by one of the authors in [11].

Type IIB model with $(h_{11}, h_{21}) = (1, 1)$ and consistent with the asymmetric nature of the shift-orbifolds presented in section 4. We also analyze a simple instance of an unoriented model with open strings. Finally, section 7 contains our conclusions and some perspectives on the issue of moduli stabilization. Useful formulas are reported in appendices A and B.

2 Twists and shifts

In view of moduli stabilization, a particularly promising class of solvable models are asymmetric orbifolds of tori [6]. Indeed, chiral twists tend to freeze out untwisted moduli while (non-geometric) shifts tend to eliminate twisted moduli. In Left-Right asymmetric constructions level matching constraints are very demanding and the perspective of a systematic analysis are daunting. A very simple class of solvable models which are equivalent to asymmetric orbifolds of special tori are free fermionic models [4, 5]. The rules for constructing modular invariant partition functions compatibly with both world-sheet and space-time supersymmetry are well understood and will be reviewed in the next section. Here we would like to offer a geometric interpretation of the free fermion \mathbb{Z}_2 -reflections in terms of T-duality twists and shifts.

We will denote by I_i a \mathbb{Z}_{2L} chiral reflection of the i^{th} Left-moving internal bosonic and fermionic coordinates

$$I_i : X_L^i \rightarrow -X_L^i, \quad X_R^i \rightarrow X_R^i, \quad \psi^i \rightarrow -\psi^i, \quad \tilde{\psi}^i \rightarrow \tilde{\psi}^i. \quad (2.1)$$

In a similar way one defines the Right-moving twist as

$$\bar{I}_i : X_L^i \rightarrow X_L^i, \quad X_R^i \rightarrow -X_R^i, \quad \psi^i \rightarrow \psi^i, \quad \tilde{\psi}^i \rightarrow -\tilde{\psi}^i. \quad (2.2)$$

In addition, we denote by $I_{i_1 i_2 \dots} = I_{i_1} I_{i_2} \dots$ the simultaneous reflections along the $(i_1 i_2 \dots)$ directions and similarly for the Right moving ones. We will consider \mathbb{Z}_2^4 orbifolds with generators including Left and Right twists I_{3456}, I_{1256} and $\bar{I}_{3456}, \bar{I}_{1256}$ respectively. Each twist breaks half of the Left or Right moving supersymmetries and one is left with 1/4 of the original spacetime susy. Moreover, all untwisted NS-NS moduli fields

$$|i\rangle_L \otimes |j\rangle_R = \psi_{-\frac{1}{2}}^i |0\rangle_L \otimes \tilde{\psi}_{-\frac{1}{2}}^j |0\rangle_R \quad i = 1, \dots, 6, \quad (2.3)$$

are projected out by the orbifold group. This implies that both shape and size deformations of the internal manifold are frozen out. Similarly, in the untwisted R-R sector one can see that only the scalar and the axion that together with the dilaton/axion NS-NS moduli complete the universal hypermultiplet survive the projection.

Let us now consider moduli coming from the twisted sector. In order to lift as many massless twisted states as possible one has to combine chiral twists with chiral (non-geometric) shifts. We denote the Left moving chiral shift along the i^{th} direction by

$$\sigma_i : X_L^i \rightarrow X_L^i + \delta, \quad X_R^i \rightarrow X_R^i; \quad (2.4)$$

with 2δ a chiral lattice vector. Similarly we denote by

$$\bar{\sigma}_i : X_R^i \rightarrow X_R^i + \bar{\delta}, \quad X_L^i \rightarrow X_L^i; \quad (2.5)$$

the Right moving shifts and by $\sigma_{i_1 i_2 \dots}, \bar{\sigma}_{i_1 i_2 \dots}$ the multiple shifts. Level matching, *i.e.* modular invariance, puts severe constraints on the allowed choices of σ 's.

Another tool one can resort to in order to eliminate twisted moduli is the judicious choice of discrete torsion [22, 23], *i.e.* of the relative signs (for \mathbb{Z}_2) that multiply orbits of amplitudes not connected by modular transformations. In the simplest case, discrete torsion relates the diagonal modular invariant to the charge conjugation one. More generally, exotic modular invariant combinations of the chiral characters can change and in some cases drastically reduce the number of massless combinations.

3 Free fermions versus asymmetric orbifolds

In order to perform a systematic search for models with few moduli in a full-fledged string description we resort to the free fermionic construction pioneered by Kawai, Lewellen and Tye [4] and by Antoniadis, Bachas and Kounnas [5].

In this description, one fermionizes the internal Left-moving bosonic coordinates

$$\partial X^i = y^i w^i \quad i = 1, \dots, 6 \quad , \quad (3.1)$$

and rewrites the worldsheet supercurrent as²

$$G = \psi^\mu \partial X_\mu + \psi^i y^i w^i, \quad \mu = 7, 8 \quad . \quad (3.2)$$

All fermions $\{\psi^\mu, \psi^i, y^i, w^i\}$ are taken to be periodic to start with. The Right-moving fermions $\{\tilde{\psi}^\mu, \tilde{\psi}^i, \tilde{y}^i, \tilde{w}^i\}$ are introduced in a similar way.

Now, let us consider the orbifolding of the free fermion system by \mathbb{Z}_2 reflections. A reflection is denoted by a fermion set b_α that includes all fermions odd under the \mathbb{Z}_2 . Spacetime susy and modular invariance put additional constraints on the allowed fermion sets. Preservation of the worldsheet supercurrent under parallel transport requires

$$\begin{aligned} \forall i \quad \# \psi^i - \# y^i - \# w^i &= 0 \pmod{2} ; \\ \forall i \quad \# \tilde{\psi}^i - \# \tilde{y}^i - \# \tilde{w}^i &= 0 \pmod{2} . \end{aligned} \quad (3.3)$$

Modular invariance (or level matching) amounts to the following conditions on the basis fermionic sets:

$$\begin{aligned} n(b_\alpha) &= 0 \pmod{8} ; \\ n(b_\alpha \cap b_\beta) &= 0 \pmod{4} ; \\ n(b_\alpha \cap b_\beta \cap b_\gamma) &= 0 \pmod{2} ; \\ n(b_\alpha \cap b_\beta \cap b_\gamma \cap b_\sigma) &= 0 \pmod{2} ; \end{aligned} \quad (3.4)$$

with $n(b)$ denoting the difference between the number of Left- and Right- moving fermions in the set b and the greek indices running over the generators of the orbifold group.

²Other choices are possible.

The free fermion description of Type IIB on the T^6 maximal torus of $SO(12)$ is obtained by including the following fermionic sets

$$\begin{aligned} F &= \{\psi^{1\dots 8} y^{1\dots 6} w^{1\dots 6} | \tilde{\psi}^{1\dots 8} \tilde{y}^{1\dots 6} \tilde{w}^{1\dots 6}\}, \\ S &= \{\psi^{1\dots 8}\}, \quad \tilde{S} = \{\tilde{\psi}^{1\dots 8}\}. \end{aligned} \quad (3.5)$$

Indeed, the quotient by F results into a sum over all possible boundary conditions of worldsheet fermions, while S and \tilde{S} realize the Left and Right moving GSO projections, ensuring spacetime susy. Omitting the integral over moduli space, the resulting partition function can be written as

$$\begin{aligned} \mathcal{T}_{4_L+4_R} &= \frac{1}{\eta^2 \bar{\eta}^2} |V_8 - S_8|^2 (|O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2) \\ &= \frac{1}{8} \left| \frac{\vartheta_3^4}{\eta^{12}} - \frac{\vartheta_4^4}{\eta^{12}} - \frac{\vartheta_2^4}{\eta^{12}} \right|^2 (|\vartheta_2|^{12} + |\vartheta_3|^{12} + |\vartheta_4|^{12}), \end{aligned} \quad (3.6)$$

where the subscript $4_L + 4_R$ reminds that 4_L and 4_R susy comes from the Left and Right movers, respectively. We write partition functions both in terms of characters of $SO(n)$ at level one or in terms of theta functions, as convenient in the specific context (see appendix A for definitions and conventions). Moreover, signs (discrete torsion) will be chosen judiciously and respecting the spin-statistic relation.

Another choice consists of keeping only the sets F and S , finding the $4_L + 0_R$ partition function

$$\begin{aligned} \mathcal{T}_{4_L+0_R} &= \frac{1}{\eta^2 \bar{\eta}^2} (V_8 - S_8) [O_{12} \bar{V}_{20} + V_{12} \bar{O}_{20} - S_{12} \bar{S}_{20} - C_{12} \bar{C}_{20}] \\ &= \frac{1}{4 \eta^{12} \bar{\eta}^{12}} (\vartheta_3^4 - \vartheta_4^4 - \vartheta_2^4) (\vartheta_3^6 \bar{\vartheta}_3^{10} - \vartheta_4^6 \bar{\vartheta}_4^{10} - \vartheta_2^6 \bar{\vartheta}_2^{10}). \end{aligned} \quad (3.7)$$

In the following, we will consider asymmetric \mathbb{Z}_2 orbifolds of the $\mathcal{N} = 4_L + 4_R$ and $\mathcal{N} = 4_L + 0_R$ models. The \mathbb{Z}_2 elements will be built out of chiral reflections I_i, \bar{I}_i and shifts $\sigma_i, \bar{\sigma}_i$. In the fermionic language twists and shifts correspond to the following actions on the worldsheet fermions

$$\begin{aligned} I_i &: \quad \psi^i \rightarrow -\psi^i, & y^i &\rightarrow -y^i; \\ \sigma_i &: \quad y^i \rightarrow -y^i, & w^i &\rightarrow -w^i; \end{aligned} \quad (3.8)$$

with identical expressions for \bar{I}_i and $\bar{\sigma}_i$ with fermions replaced by tilde ones. Alternatively, one can denote reflections and shifts by their associated fermionic sets

$$\begin{aligned} I_i &= \{\psi^i y^i\}, & \sigma_i &= \{y^i w^i\}, \\ \bar{I}_i &= \{\tilde{\psi}^i \tilde{y}^i\}, & \bar{\sigma}_i &= \{\tilde{y}^i \tilde{w}^i\}. \end{aligned} \quad (3.9)$$

Notice that the non-trivial intersections between the sets are

$$\begin{aligned} I_i \cap I_j &= \sigma_i \cap \sigma_j = 2 \delta_{ij}, & I_i \cap \sigma_j &= \delta_{ij}, \\ \bar{I}_i \cap \bar{I}_j &= \bar{\sigma}_i \cap \bar{\sigma}_j = 2 \delta_{ij}, & \bar{I}_i \cap \bar{\sigma}_j &= \delta_{ij}. \end{aligned} \quad (3.10)$$

These relations can be used to check the consistency conditions (3.4) of an orbifold group generated by twists and shifts.

4 Models with $\mathcal{N} = 1_L + 1_R$

We performed a systematic search of models with basis sets F, S, \tilde{S} together with four additional sets of the form

$$\begin{aligned}
 b_1 &= (b_{1L}, b_{1R}) = I_{3456} \sigma^{i_1 i_2 \dots} \bar{\sigma}^{k_1 k_2 \dots} = \left\{ (\psi y)^{3456} (y w)^{i_1 i_2 \dots} | (\tilde{y} \tilde{w})^{k_1 k_2 \dots} \right\}, \\
 b_2 &= (b_{2L}, b_{2R}) = I_{1256} \sigma^{j_1 j_2 \dots} \bar{\sigma}^{l_1 l_2 \dots} = \left\{ (\psi y)^{1256} (y w)^{j_1 j_2 \dots} | (\tilde{y} \tilde{w})^{l_1 l_2 \dots} \right\}, \\
 \bar{b}_1 &= (b_{1R}, b_{1L}) = \bar{I}_{3456} \sigma^{k_1 k_2 \dots} \bar{\sigma}^{i_1 i_2 \dots} = \left\{ (y w)^{k_1 k_2 \dots} | (\tilde{\psi} \tilde{y})^{3456} (\tilde{y} \tilde{w})^{i_1 i_2 \dots} \right\}, \\
 \bar{b}_2 &= (b_{2R}, b_{2L}) = \bar{I}_{1256} \sigma^{l_1 l_2 \dots} \bar{\sigma}^{j_1 j_2 \dots} = \left\{ (y w)^{l_1 l_2 \dots} | (\tilde{\psi} \tilde{y})^{1256} (\tilde{y} \tilde{w})^{j_1 j_2 \dots} \right\}, \quad (4.1)
 \end{aligned}$$

The scanning ran over all choices of sets $(i_1 i_2 \dots)$, $(j_1 j_2 \dots)$, $(k_1 k_2 \dots)$, $(l_1 l_2 \dots)$ compatibly with the conditions (3.4). Each set b_α breaks half of the spacetime susy's arising from the Left- or Right- moving sector. One is thus left with $\mathcal{N} = 1_L + 1_R$ susy.

Defining for notational convenience³ $b_3 = b_1 b_2$, $\bar{b}_3 = \bar{b}_1 \bar{b}_2$, the generic orbifold group element can be written as $b_a \bar{b}_b$ with $a, b = 0, \dots, 3$ and $b_0 = \bar{b}_0 = 1$. We recall that a contribution of a single Left moving fermion among $\{\psi^i, y^i, w^i\}$ is given by $(\vartheta_s/\eta)^{\frac{1}{2}}$ (with $s = 2, 3, 4$ labelling the spin structure), and similarly for Right moving fermions with ϑ_s/η replaced by $\bar{\vartheta}_s/\bar{\eta}$. The \mathbb{Z}_2 actions are thus equivalent to

$$\mathbb{Z}_2 : \vartheta_2^{\frac{1}{2}} \rightarrow \vartheta_1^{\frac{1}{2}}, \quad \vartheta_3^{\frac{1}{2}} \leftrightarrow \vartheta_4^{\frac{1}{2}}, \quad \bar{\vartheta}_2^{\frac{1}{2}} \rightarrow \bar{\vartheta}_1^{\frac{1}{2}}, \quad \bar{\vartheta}_3^{\frac{1}{2}} \leftrightarrow \bar{\vartheta}_4^{\frac{1}{2}}. \quad (4.2)$$

The torus partition function can then be written as

$$\mathcal{T} = \frac{1}{16 \eta^8 \bar{\eta}^8} \sum_{a,b,c,d=0}^3 \rho_{ac} \bar{\rho}_{bd} \Lambda \left[\begin{matrix} ab \\ cd \end{matrix} \right], \quad (4.3)$$

where $\Lambda \left[\begin{matrix} ab \\ cd \end{matrix} \right]$ denotes the contribution of the $(b_c \bar{b}_d)$ -projection in the $(b_a \bar{b}_b)$ -twisted sector, i.e.

$$\Lambda \left[\begin{matrix} ab \\ cd \end{matrix} \right] = \frac{1}{2} \epsilon_{a,b,c,d} \sum_{\alpha, \beta=0, \frac{1}{2}}^{12} \prod_{i=1}^{12} \vartheta \left[\begin{matrix} \alpha + b_{aL,i} + b_{bR,i} \\ \beta + b_{cL,i} + b_{dR,i} \end{matrix} \right]^{\frac{1}{2}} \bar{\vartheta} \left[\begin{matrix} \alpha + b_{aR,i} + b_{bL,i} \\ \beta + b_{cR,i} + b_{dL,i} \end{matrix} \right]^{\frac{1}{2}} \quad (4.4)$$

with i running over the 12 lattice fermions $y^i w^i$ and $b_{La,i}, b_{Ra,i}$ being 0 or $\frac{1}{2}$ depending on whether the i^{th} fermion is even or odd under b_{aL} and b_{aR} respectively. $\epsilon_{a,b,c,d}$ are signs fixed by modular invariance up to discrete torsions. The precise form of $\Lambda \left[\begin{matrix} ab \\ cd \end{matrix} \right]$ depends on the details of the specific model. Finally the ψ -contribution to the amplitudes can be

³The product is defined as $b_i b_j = b_i \cup b_j - b_i \cap b_j$

written as

$$\begin{aligned}
\rho_{00} &= -\frac{\vartheta_{2,\text{st}}\vartheta_2^3 - \vartheta_{3,\text{st}}\vartheta_3^3 + \vartheta_{4,\text{st}}\vartheta_4^3}{2\eta^4}, \\
\rho_{0h} &= \frac{\vartheta_{3,\text{st}}\vartheta_3\vartheta_4^2 - \vartheta_{4,\text{st}}\vartheta_4\vartheta_3^2}{2\eta^4}, \\
\rho_{h0} &= \frac{\vartheta_{3,\text{st}}\vartheta_3\vartheta_2^2 - \vartheta_{2,\text{st}}\vartheta_2\vartheta_3^2}{2\eta^4}, \\
\rho_{hh} &= \frac{\vartheta_{2,\text{st}}\vartheta_2\vartheta_4^2 - \vartheta_{4,\text{st}}\vartheta_4\vartheta_2^2}{2\eta^4}, \\
\rho_{13} = \rho_{23} = \rho_{32} = -\rho_{12} = -\rho_{21} = -\rho_{31} &= \frac{i\vartheta_{1,\text{st}}\vartheta_2\vartheta_3\vartheta_4}{2\eta^4}, \tag{4.5}
\end{aligned}$$

with $h = 1, 2, 3$ and the subscript “st” denoting the contribution coming from the spacetime part that encodes the helicity of the particle (see appendix A for the definitions of the amplitudes given in terms of the $\text{SO}(2n)$ characters). The massless content of each model can be read by plugging in the partition function the well-known theta expansions

$$\begin{aligned}
\vartheta_{1,\text{st}} &= (S - C)q^{\frac{1}{8}} + \dots, & \vartheta_{2,\text{st}} &= (S + C)q^{\frac{1}{8}} + \dots, \\
\vartheta_{3,\text{st}} &= 1 + Vq^{\frac{1}{2}} + \dots, & \vartheta_{4,\text{st}} &= 1 - Vq^{\frac{1}{2}} + \dots, \\
\vartheta_2 &= 2q^{\frac{1}{8}} + \dots, & \vartheta_{3,4} &= 1 + 2q^{\frac{1}{2}} + \dots, & \eta &= q^{\frac{1}{24}} + \dots. \tag{4.6}
\end{aligned}$$

where O, V, S, C denote a four-dimensional scalar, vector, left spinor and right spinor, respectively. The result can always be written in the form

$$\begin{aligned}
\mathcal{T}_0 &= |V - S - C|^2 + n_v [|O - S|^2 + |O - C|^2] \\
&\quad + (n_h - 1) [(O - S)(\bar{O} - \bar{C}) + (O - C)(\bar{O} - \bar{S})] + \dots \\
&= \mathbf{G}_2 + n_v \mathbf{V}_2 + n_h \mathbf{H}_2, \tag{4.7}
\end{aligned}$$

with n_h and n_v the number of hyper- and vector-multiplets respectively, and

$$\begin{aligned}
\mathbf{G}_2 + \mathbf{H}_2 &= |V - S - C|^2, \\
\mathbf{V}_2 &= |O - S|^2 + |O - C|^2, \\
\mathbf{H}_2 &= (O - S)(\bar{O} - \bar{C}) + (O - C)(\bar{O} - \bar{S}) \tag{4.8}
\end{aligned}$$

the $\mathcal{N} = 2$ supergravity, hyper- and vector-multiplet contents, each comprising $4_B + 4_F$ physical degrees of freedom. Due to the asymmetric twists and shifts, the resulting vacuum configurations do not correspond to compactifications of Type IIB on geometric CY manifolds, yet the theory enjoys $\mathcal{N} = 2$ spacetime susy. We are thus led to define the “effective” Hodge numbers

$$h_{11} = n_h - 1, \quad h_{21} = n_v, \tag{4.9}$$

and also to define the “effective” Euler characteristic $\chi = 2(h_{11} - h_{21}) = 2(n_h - n_v) - 2$.

In the following, we first describe in some details the simplest model with minimal massless content,⁴ namely the one with $(h_{11}, h_{12}) = (1, 1)$. Then, we report the complete list of models resulting from our scan.

4.1 An example: $(h_{11}, h_{12}) = (1, 1)$

One of the possible choices of twists and shifts that give rise to an interesting $(h_{11}, h_{12}) = (1, 1)$ is the following:

$$\begin{aligned}
 b_1 &= I_{3456} \sigma_1 \bar{\sigma}_5, \\
 b_2 &= I_{1256} \sigma_3 \bar{\sigma}_{12345}, \\
 \bar{b}_1 &= \bar{I}_{3456} \sigma_5 \bar{\sigma}_1, \\
 \bar{b}_2 &= \bar{I}_{1256} \sigma_{12345} \bar{\sigma}_3.
 \end{aligned} \tag{4.10}$$

Many of the amplitudes vanish due to the presence of $SO(12)$ fermions in the odd spin structure. The lattice sums of the non-vanishing amplitudes read

$$\begin{aligned}
 \Lambda_{[00]}^{[00]} &= \frac{1}{2} (|\vartheta_2|^{12} + |\vartheta_3|^{12} + |\vartheta_4|^{12}) \\
 \Lambda_{[h0]}^{[00]} &= \frac{1}{2} \vartheta_3^3 \vartheta_4^3 \bar{\vartheta}_3 \bar{\vartheta}_4 (\bar{\vartheta}_3^4 + \bar{\vartheta}_4^4) \\
 \Lambda_{[30]}^{[00]} &= \frac{1}{2} |\vartheta_3 \vartheta_4|^4 (\vartheta_4^2 \bar{\vartheta}_3^2 + \vartheta_3^2 \bar{\vartheta}_4^2) \\
 \Lambda_{[hh']}^{[00]} &= \Lambda_{[h3]}^{[00]} = |\vartheta_3 \vartheta_4|^6 \\
 \Lambda_{[33]}^{[00]} &= \frac{1}{2} |\vartheta_3 \vartheta_4|^4 (|\vartheta_3|^2 + |\vartheta_4|^2) \\
 \Lambda_{[h0]}^{[h0]} &= \frac{1}{2} \vartheta_2^3 \vartheta_3^3 \bar{\vartheta}_2 \bar{\vartheta}_3 (\bar{\vartheta}_2^4 + \bar{\vartheta}_3^4) \\
 \Lambda_{[30]}^{[30]} &= \frac{1}{2} |\vartheta_2 \vartheta_3|^4 (\vartheta_3^2 \bar{\vartheta}_2^2 + \vartheta_2^2 \bar{\vartheta}_3^2) \\
 \Lambda_{[hh']}^{[hh']} &= \Lambda_{[h3]}^{[h3]} = |\vartheta_2 \vartheta_3|^6 \\
 \Lambda_{[33]}^{[33]} &= \frac{1}{2} |\vartheta_2 \vartheta_3|^4 (|\vartheta_2|^4 + |\vartheta_3|^4) \\
 \Lambda_{[h0]}^{[h0]} &= \vartheta_2^3 \vartheta_4^3 \bar{\vartheta}_2 \bar{\vartheta}_4 (\bar{\vartheta}_2^4 - \bar{\vartheta}_4^4) \\
 \Lambda_{[30]}^{[30]} &= \frac{1}{2} |\vartheta_2 \vartheta_4|^4 (\vartheta_2^2 \bar{\vartheta}_4^2 - \vartheta_4^2 \bar{\vartheta}_2^2) \\
 \Lambda_{[hh']}^{[hh']} &= |\vartheta_2 \vartheta_4|^6 \\
 \Lambda_{[33]}^{[33]} &= \frac{1}{2} |\vartheta_2 \vartheta_4|^4 (|\vartheta_2|^4 + |\vartheta_4|^4)
 \end{aligned} \tag{4.11}$$

with $h, h' = 1, 2$ and $\Lambda_{[cd]}^{[ab]} = \Lambda_{[dc]}^{[ba]}^*$. Thanks to the four independent \mathbb{Z}_2 chiral twists all massless states in the untwisted sector, except the $\mathcal{N} = 2$ supergravity multiplet and the universal dilaton hypermultiplet, are projected out. In addition, due to the chiral shifts,

⁴A related but different model with extended $\mathcal{N} = 2_L + 2_R$ susy and thus larger massless multiplets has been exhibited in [20].

most twisted sectors, except for the (3, 3) sector, contribute only massive states. Indeed, a twisted sector contribute massless states only when the shifts are along the reflection plane. This condition is satisfied only for $b_3 \bar{b}_3 = I_{1234} \bar{I}_{1234} \sigma_{24} \bar{\sigma}_{24}$.

As a consequence, the only massless contributions are:

$$\sum_{a,b=0}^3 \mathcal{T}_0 \begin{bmatrix} 00 \\ ab \end{bmatrix} = |V - S - C|^2, \\ \mathcal{T}_0 \begin{bmatrix} 33 \\ 00 \end{bmatrix} + \mathcal{T}_0 \begin{bmatrix} 33 \\ 33 \end{bmatrix} = |2O - S - C|^2. \tag{4.12}$$

The latter, as anticipated, gives precisely one hyper- and one vector- multiplet. This is the minimal massless content among all known Type II compactifications admitting an isomorphism under exchange of Left- and Right-movers and thus amenable to a natural unoriented projection. We remark that several Left-Right asymmetric models are known with fewer moduli: for instance, a “minimal” $\mathcal{N} = 2_L + 0_R$ model with only the dilaton vector multiplet recently exhibited in [24] as a starting point for the construction of “magic” $\mathcal{N} = 2$ supergravity theories [25]. Moreover, there are models with $\mathcal{N} = 3$ susy constructed in [26] with only one vector multiplet, comprising only 3 complex massless scalars, including the dilaton. Other systematic searches of models with low “effective” Hodge numbers [21] seem to only focus on Left-Right symmetric twists and shifts that lead at most (or at least) to $h_{11} = h_{21} = 3$, well known from the work of Vafa and Witten [23] and more recently of [27] in the realm of Type I/heterotic duality.

4.2 Various $\mathcal{N} = 1_L + 1_R$ models

In addition to the above model, we have found many (new) Type IIB non-geometric yet Left-Right symmetric models with low “effective” Hodge numbers, which may still turn out to be interesting starting points for Type I model building. We remark that although our models are given in terms of a rational CFT, a systematic study of open string descendants is complicated by the high number of characters involved, typically of the order of one thousand. They can probably be explored by computer means along the lines of [20].

In table 1 we report all our consistent models. We keep track of the pattern of (pseudo)symmetry breaking $SO(12) \rightarrow \prod_I SO(n_I)$. Curiously, the whole list of models can be grouped into the following three finite series for (h_{11}, h_{12}) :

$$\begin{aligned} (n, n) & \quad n = 1, 2, 3, 4, 5, 9 \\ (2n, 2n + 6), (2n + 6, 2n) & \quad n = 0, 1, 2 \\ (2n + 3, 2n + 15), (2n + 15, 2n + 3) & \quad n = 0, 1 \end{aligned} \tag{4.13}$$

Our systematic search scans over all possible shifts with all discrete torsion signs taken to be plus. A longer list of consistent models can be built after playing with more general discrete torsion choices. In particular, it should be noticed that the “effective” Euler number is always a multiple of 12, as claimed in the Introduction. Finally, as apparent from table 1, identical patterns of (pseudo)symmetry breaking may lead to rather different massless spectra. This can be explained, in the cases under consideration, by noticing that models with the same breaking differ due to different choices of discrete torsions.

The results of our systematic search partly overlap with the recent results of a scan over \mathbb{Z}_2 orbifolds of the product of 18 Ising models presented in [20]. An important difference between the two searches is the choice of the T^6 lattice. We start with the non-factorizable $\text{SO}(12)$ maximal torus $T_{\text{SO}(12)}^6$ while the authors of [20] start with a factorizable T^6 .

b's	$\text{SO}(12)$	(h_{11}, h_{12})
$I_{3456} \sigma_1 \bar{\sigma}_5$ $I_{1256} \sigma_3 \bar{\sigma}_{12345}$ $\bar{I}_{3456} \sigma_5 \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_{12345} \bar{\sigma}_3$	$\text{SO}(2)^4 \times O(1)^4$	(1,1)
$I_{3456} \sigma_1 \bar{\sigma}_2$ $I_{1256} \sigma_3 \bar{\sigma}_{12345}$ $I_{3456} \sigma_2 \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_{12345} \bar{\sigma}_3$	$\text{SO}(3) \times \text{SO}(2)^2 \times O(1)^5$	(2,2)
$I_{3456} \sigma_{12} \bar{\sigma}_{123456}$ $I_{1256} \sigma_{236} \bar{\sigma}_1$ $\bar{I}_{3456} \sigma_{123456} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_1 \bar{\sigma}_{236}$	$\text{SO}(3)^2 \times \text{SO}(2)^2 \times O(1)^2$	(3,3)
$I_{3456} \sigma_1 \bar{\sigma}_5$ $I_{1256} \sigma_3 \bar{\sigma}_{12456}$ $\bar{I}_{3456} \sigma_5 \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_{12456} \bar{\sigma}_3$	$\text{SO}(3) \times \text{SO}(2)^2 \times O(1)^5$	(4,4)
$I_{3456} \sigma_{126} \bar{\sigma}_{12}$ $I_{1256} \sigma_{346} \bar{\sigma}_{35}$ $\bar{I}_{3456} \sigma_{12} \bar{\sigma}_{126}$ $\bar{I}_{1256} \sigma_{35} \bar{\sigma}_{346}$	$\text{SO}(2)^4 \times O(1)^4$	(5,5)
$I_{3456} \sigma_{12} \bar{\sigma}_{12}$ $I_{1256} \sigma_{34} \bar{\sigma}_{56}$ $\bar{I}_{3456} \sigma_{12} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_{56} \bar{\sigma}_{34}$	$\text{SO}(2)^6$	(9,9)
$I_{3456} \sigma_{12} \bar{\sigma}_{13}$ $I_{1256} \sigma_{34} \bar{\sigma}_{25}$ $\bar{I}_{3456} \sigma_{13} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_{25} \bar{\sigma}_{34}$	$\text{SO}(2)^3 \times O(1)^6$	(6,0)
$I_{3456} \sigma_{12} \bar{\sigma}_{15}$ $I_{1256} \sigma_{34} \bar{\sigma}_{36}$ $\bar{I}_{3456} \sigma_{15} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_{36} \bar{\sigma}_{34}$	$\text{SO}(2)^3 \times O(1)^6$	(0,6)
$I_{3456} \sigma_1 \bar{\sigma}_4$ $I_{1256} \sigma_{356} \bar{\sigma}_2$ $\bar{I}_{3456} \sigma_4 \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_2 \bar{\sigma}_{356}$	$\text{SO}(3)^2 \times \text{SO}(2) \times O(1)^4$	(2,8)

Table 1 — continued on next page

b's	SO(12)	(h_{11}, h_{12})
$I_{3456} \sigma_1 \bar{\sigma}_2$ $I_{1256} \sigma_{356} \bar{\sigma}_4$ $\bar{I}_{3456} \sigma_2 \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_4 \bar{\sigma}_{356}$	$SO(3)^2 \times SO(2) \times O(1)^4$	(8,2)
$I_{3456} \sigma_1 \bar{\sigma}_5$ $I_{1256} \sigma_{346} \bar{\sigma}_{25}$ $\bar{I}_{3456} \sigma_5 \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_{25} \bar{\sigma}_{346}$	$SO(3)^2 \times SO(2) \times O(1)^4$	(4,10)
$I_{3456} \sigma_{12} \bar{\sigma}_{45}$ $I_{1256} \sigma_{36} \bar{\sigma}_5$ $\bar{I}_{3456} \sigma_{45} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_5 \bar{\sigma}_{36}$	$SO(3)^2 \times SO(2) \times O(1)^4$	(10,4)
$I_{3456} \sigma_{12} \bar{\sigma}_{12}$ $I_{1256} \sigma_{34} \bar{\sigma}_{34}$ $\bar{I}_{3456} \sigma_{12} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_{34} \bar{\sigma}_{34}$	$SO(2)^6$	(15,3)
$I_{3456} \bar{\sigma}_{3456}$ $I_{1256} \bar{\sigma}_{1256}$ $\bar{I}_{3456} \sigma_{3456}$ $\bar{I}_{1256} \sigma_{1256}$	$SO(2)^6$	(3,15)
$I_{3456} \sigma_{12} \bar{\sigma}_{34}$ $I_{1256} \sigma_{34} \bar{\sigma}_{123456}$ $\bar{I}_{3456} \sigma_{34} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_{123456} \bar{\sigma}_{34}$	$SO(4) \times SO(2)^4$	(5,17)
$I_{3456} \sigma_{126} \bar{\sigma}_{123456}$ $I_{1256} \sigma_5 \bar{\sigma}_{3456}$ $\bar{I}_{3456} \sigma_{123456} \bar{\sigma}_{12}$ $\bar{I}_{1256} \sigma_{3456} \bar{\sigma}_5$	$SO(4) \times SO(2)^4$	(17,5)
$I_{3456} \sigma_1 \bar{\sigma}_{12456}$ $I_{1256} \sigma_{356} \bar{\sigma}_{23456}$ $\bar{I}_{3456} \sigma_{12456} \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_{23456} \bar{\sigma}_{356}$	$SO(3)^2 \times SO(2) \times O(1)^4$	(6,12)
$I_{3456} \sigma_1 \bar{\sigma}_{23456}$ $I_{1256} \sigma_{356} \bar{\sigma}_{12456}$ $\bar{I}_{3456} \sigma_{23456} \bar{\sigma}_1$ $\bar{I}_{1256} \sigma_{12456} \bar{\sigma}_{356}$	$SO(3)^2 \times SO(2) \times O(1)^4$	(12,6)

Table 1: Inequivalent Type II models with $\mathcal{N} = 1_L + 1_R$ supersymmetry resulting from asymmetric Z_2^4 orbifold projections on the $SO(12)$ maximal torus. The first column contains the four generators of the orbifold group. The second column, the 'fake' global symmetry surviving to the breaking of $SO(12)$. The third column, the effective Hodge numbers.

5 Models with $\mathcal{N} = 1_L$

Another class of interesting Type II models are the Left-Right asymmetric orbifolds with $\mathcal{N} = 1_L$ spacetime susy. In the bosonic description, these models arise from including a projection $(-)^{F_R}\sigma_R$, thus breaking all supersymmetries associated to the Right-movers and preventing any of those to reappear in the twisted sectors by means of the order two chiral shift σ_R . In the fermionic description, σ_R simply amount to a reflection of all the SO(12) fermions. Thus, the projection is equivalent to choose a basis of sets consisting only of F and S . This is the starting point of our systematic search in this largely unexplored class of Type II vacuum configurations. The resulting $\mathcal{N} = 4_L + 0_R$ spectrum is coded in the one-loop torus amplitude (3.7). Supersymmetric massless states only arise from the combination $(V_8 - S_8)O_{12}\bar{V}_{20}$, that produces $\mathcal{N} = 4$ supergravity coupled to 18 vector multiplets. A careful look at the corresponding vertex operators and their OPE's shows that the gauge group is $SU(2)^6$, as a remnant of the structure of the internal world-sheet cubic supercurrent [29]. This or an equivalent model has been found in the seminal paper [28]. The emergence of Right-moving world-sheet currents, generating a supersymmetric Kac-Moody algebra, has been deeply analyzed in view of the possibility of producing non-abelian NS gauge symmetries. The authors of [28] arrived however at the negative conclusion that (perturbative) Type II models cannot accommodate the Standard Model with its matter content.

To the sets F and S we have added two more sets b_1 and b_2 producing a breaking of spacetime susy down to $\mathcal{N} = 1_L + 0_R$ and, at the same time, a breaking of the internal (pseudo)symmetry SO(20). Indeed, what we said in the context of the above $\mathcal{N} = 4_L + 0_R$ model applies to $\mathcal{N} = 1_L + 0_R$, too. The “true” gauge symmetry can only be determined after a careful analysis of the vertex operators for the vector fields and their OPE's, while taking into account the precise structure of the cubic supercurrent. Since the only cubic supercurrent we consider is expressed in terms of the $SU(2)^6$ structure constants, the resulting gauge symmetry we find is a subgroup of $SU(2)^6$ with abelian factors. Moreover, there are massless charged chiral multiplets that can further break the gauge symmetry by a perturbative Higgs mechanism.

In the following we describe in some details a specific model with minimal number of chiral multiplets and then collect the remaining models in table 2. The massless spectrum decomposes according to

$$\mathcal{T}_0 = \mathbf{G}_1 + n_v \mathbf{V}_1 + n_{v'} \mathbf{V}'_1 + n_c \mathbf{C}_1 + n_{c'} \mathbf{C}'_1, \tag{5.1}$$

with

$$\begin{aligned} \mathbf{G}_1 + \mathbf{C}_1 &= (V - S - C) \bar{V}, \\ \mathbf{V}_1 &= (V - S - C) \bar{O} \\ \mathbf{V}'_1 &= S\bar{S} + C\bar{C} - S\bar{O} - C\bar{O}, \\ \mathbf{C}_1 &= (2O - S - C) \bar{O} \\ \mathbf{C}'_1 &= C\bar{S} + S\bar{C} - O\bar{S} - O\bar{C} \end{aligned} \tag{5.2}$$

the content of the gravity, vector and chiral multiplets, and $n_v + n_{v'}$, $n_c + n_{c'}$ the total numbers of vector and chiral multiplets. Although primed and unprimed multiplets have identical field content, we find it convenient to distinguish them in order to stress the

different origin, NS-NS or R-R, of their bosonic degrees of freedom. It is amusing to stress that generalized D-branes [11] and their exotic open string excitations can be introduced that couple to the twisted R-R states.

5.1 An example: $(n_v, n'_v; n_c, n'_c) = (14, 0; 5, 0)$

Let us discuss the model with generators

$$\begin{aligned} b_1 &= I_{3456} \sigma_{12} \bar{\sigma}_{45} , \\ b_2 &= I_{1256} \sigma_{36} \bar{\sigma}_5 . \end{aligned} \tag{5.3}$$

In addition to breaking spacetime supersymmetry to $\mathcal{N} = 1$, the two \mathbb{Z}_2 actions break the internal (pseudo)symmetry according to

$$\text{SO}(12)_L \times \text{SO}(20)_R \rightarrow [\text{SO}(4)^2 \times \text{SO}(2)^2]_L \times [\text{SO}(2)^2 \times \text{SO}(16)]_R . \tag{5.4}$$

Actually, $\text{SO}(16)_R \rightarrow \text{SO}(2) \times \text{SO}(14)$, where the first factor is the little group for massless particles in $D = 4$.

The non-vanishing lattice sums read

$$\begin{aligned} \Lambda [0] &= \frac{1}{2} (\vartheta_3^6 \bar{\vartheta}_3^{10} - \vartheta_4^6 \bar{\vartheta}_4^{10} - \vartheta_2^6 \bar{\vartheta}_2^{10}) \\ \Lambda [1] &= \frac{1}{2} \vartheta_3^2 \vartheta_4^2 \bar{\vartheta}_3^2 \bar{\vartheta}_4^2 (\vartheta_4^2 \bar{\vartheta}_3^6 - \vartheta_3^2 \bar{\vartheta}_4^6) \\ \Lambda [1] &= \frac{1}{2} \vartheta_2^2 \vartheta_3^2 \bar{\vartheta}_2^2 \bar{\vartheta}_3^2 (\vartheta_2^2 \bar{\vartheta}_3^6 - \vartheta_3^2 \bar{\vartheta}_2^6) \\ \Lambda [1] &= \frac{1}{2} \vartheta_2^2 \vartheta_4^2 \bar{\vartheta}_2^2 \bar{\vartheta}_4^2 (\vartheta_2^2 \bar{\vartheta}_4^6 - \vartheta_4^2 \bar{\vartheta}_2^6) \\ \Lambda [2] = \Lambda [3] &= \frac{1}{2} \vartheta_3^3 \vartheta_4^3 \bar{\vartheta}_3 \bar{\vartheta}_4 (\bar{\vartheta}_3^8 - \bar{\vartheta}_4^8) \\ \Lambda [2] = \Lambda [3] &= \frac{1}{2} \vartheta_2^3 \vartheta_3^3 \bar{\vartheta}_2 \bar{\vartheta}_3 (\bar{\vartheta}_3^8 - \bar{\vartheta}_2^8) \\ \Lambda [2] = \Lambda [3] &= \frac{1}{2} \vartheta_2^3 \vartheta_4^3 \bar{\vartheta}_2 \bar{\vartheta}_4 (\bar{\vartheta}_4^8 - \bar{\vartheta}_2^8) . \end{aligned} \tag{5.5}$$

Massless states come only from the untwisted sector leading to

$$\mathcal{T}_0 = (V - S - C)(\bar{V} + 14\bar{O}) + 4(2O - S - C)\bar{O} = \mathbf{G}_1 + 14\mathbf{V}_1 + 5\mathbf{C}_1 . \tag{5.6}$$

The resulting gauge group is $\text{SU}(2)^4 \times \text{U}(1)^2$. The universal chiral multiplet is neutral, while the additional four chiral multiplets are charged with respect to the abelian factors. They form two pairs of charge $(\pm 1, 0)$ and $(0, \pm 1)$. Along the flat directions of the D-term potential, the $\text{U}(1)^2$ gauge symmetry is generically broken. Since no matter fields are charged with respect to $\text{SU}(2)^6$, the latter remains as an unbroken gauge symmetry in perturbation theory. It would be very important to study the possibility of including both physical and Euclidean Left-Right asymmetric D-branes in the background in order to have a richer matter spectrum and turn on non-perturbative effects.

b's	$SO(12)_L \times SO(20)_R$	$(n_v, n_{v'}; n_c, n_{c'})$
$I_{3456} \sigma_{12} \bar{\sigma}_{45}$ $I_{1256} \sigma_{36} \bar{\sigma}_5$	$[SO(4)^2 \times SO(2)^2]_L \times [SO(16) \times SO(2)^2]_R$	(14,0;5,0)
$I_{3456} \sigma_{126} \bar{\sigma}_{12}$ $I_{1256} \sigma_{346} \bar{\sigma}_{35}$	$[SO(6) \times SO(2)^3]_L \times [SO(4)^2 \times SO(12)]_R$	(10,0;25,0)
$I_{3456} \sigma_1 \bar{\sigma}_5$ $I_{1256} \sigma_3 \bar{\sigma}_{12345}$	$[SO(4)^2 \times SO(2)^2]_L \times [SO(8) \times SO(2) \times SO(10)]_R$	(8,0;27,0)
$I_{3456} \sigma_{12} \bar{\sigma}_{123456}$ $I_{1256} \sigma_{236} \bar{\sigma}_1$	$[SO(4)^2 \times SO(2)^2]_L \times [SO(2) \times SO(10) \times SO(8)]_R$	(6,8;13,8)
$I_{3456} \sigma_{12} \bar{\sigma}_{34}$ $I_{1256} \sigma_{34} \bar{\sigma}_{123456}$	$[SO(6) \times SO(2)^3]_L \times [SO(4) \times SO(8) \times SO(8)]_R$	(6,8;29,8)

Table 2. Inequivalent Type II models with $\mathcal{N} = 1_L + 1_R$ supersymmetry resulting from asymmetric Z_2^2 orbifold projections on the $SO(12)$ maximal torus. The first column contains the two generators of the orbifold group. The second column, the 'fake' global symmetry surviving to the breaking of $SO(12)_L \times SO(20)_R$. The third column, the numbers of vector and chiral multiplets of different kinds (see main text).

5.2 Various $\mathcal{N} = 1_L$ models

Table 2 summarizes the results of our preliminary search of Type IIB models with $\mathcal{N} = 1_L + 0_R$. The basis sets are now, besides the universal F and S , the two additional

$$\begin{aligned}
 b_1 &= I_{3456} \sigma^{i_1 i_2 \dots} \bar{\sigma}^{k_1 k_2 \dots} = \left\{ (\psi y)^{3456} (y w)^{i_1 i_2 \dots} | (\tilde{y} \tilde{w})^{k_1 k_2 \dots} \right\}, \\
 b_2 &= I_{1256} \sigma^{j_1 j_2 \dots} \bar{\sigma}^{l_1 l_2 \dots} = \left\{ (\psi y)^{1256} (y w)^{j_1 j_2 \dots} | (\tilde{y} \tilde{w})^{l_1 l_2 \dots} \right\},
 \end{aligned}
 \tag{5.7}$$

with the scanning that runs over all choices of the sets $(i_1 i_2 \dots)$, $(j_1 j_2 \dots)$, $(k_1 k_2 \dots)$, $(l_1 l_2 \dots)$, compatibly again with the conditions (3.4). Each set b_α breaks half of the spacetime susy's arising from the Left-moving sector, while supersymmetry associated to the Right-moving sectors is completely broken to start with. As apparent from table 2, the reduction in the number of moduli in $\mathcal{N} = 1_L$ models is less significant than in $\mathcal{N} = 1_L + 1_R$ models. This is due to the presence of the tachyonic vacuum in the R-moving sector that, combined with internal excitations, can produce physical (*i.e.* level matched) particle states.

6 Unoriented projections

The Left-Right symmetric Type IIB string vacua we constructed in section 4 admit a natural Ω projection. It is an interesting question whether they can be taken as a starting point of orientifold constructions with phenomenologically interesting open string chiral matter. For unoriented strings [9], several closed string moduli are odd under Ω and are thus projected out. Hypermultiplets of the oriented $\mathcal{N} = 2$ theory reduce to $\mathcal{N} = 1$ chiral multiplets while vector multiplets lead to vector or chiral multiplets according to their parity under Ω . In addition, D-brane sectors should be added in the presence of non-trivial closed string tadpoles.

Here we discuss the simplest instances of unoriented projections with and without open strings.

6.1 The minimal model

We start by considering the unoriented projection of the $\mathcal{N} = 1_L + 1_R$ model with $(h_{11}, h_{21}) = (1, 1)$ discussed in section 4.1, corresponding to the choice of generators in eq. (4.10). Notice that, since Ω identifies Left and Right movers, one has

$$\text{Tr}_{\mathcal{H}_L \otimes \mathcal{H}_R} \Omega (g^L \otimes g^R) = \text{Tr}_{\mathcal{H}_L} g_\Omega, \quad (6.1)$$

where g_Ω is the diagonal action $g^L g^R$ with Left and Right moving fields identified, i.e. $\bar{I}_i \rightarrow I_i, \bar{\sigma}_i \rightarrow \sigma_i$. In this way, the g_Ω amplitudes are not the naive chiral halves of the amplitudes entering the torus amplitude. They must be rather written in terms of traces over the chiral modes of the g_Ω orbifold group generators corresponding to the sets

$$\begin{aligned} b_{1\Omega} &= I_{3456} \sigma_{15} = \{ \psi^{3456} y^{1346} w^{15} \}, \\ b_{2\Omega} &= I_{1256} \sigma_{1245} = \{ \psi^{1256} y^{46} w^{1245} \}, \\ b_{3\Omega} &= I_{1234} \sigma_{24} = \{ \psi^{1234} y^{13} w^{24} \}, \end{aligned} \quad (6.2)$$

where $b_{3\Omega} = b_{1\Omega} b_{2\Omega}$. In addition, only Left-Right symmetrically twisted states enter the Klein-bottle amplitude. In the direct channel one then gets

$$\mathcal{K} = \frac{1}{16} \sum_{a,b,c,d} \text{Tr}_{\mathcal{H}_{Lc} \otimes \mathcal{H}_{Rd}} \Omega b_a \bar{b}_b = \frac{1}{4} \sum_{a,b} \text{Tr}_{\mathcal{H}_{La}} b_{b\Omega} = \frac{1}{4 \eta^8} \sum_{a,b=0}^3 \epsilon_{a,b} \rho_{ab} \Lambda [b^a] \quad (6.3)$$

where the unoriented lattice sums read

$$\Lambda [0] = \vartheta_3^6 + \epsilon \vartheta_2^6 \quad (6.4)$$

$$\Lambda [h] = \vartheta_4^3 \vartheta_3^3 + \epsilon \vartheta_1^3 \vartheta_2^3 \quad (6.5)$$

$$\Lambda [3] = \vartheta_4^2 \vartheta_3^4 + \epsilon \vartheta_1^2 \vartheta_2^4 \quad (6.6)$$

$$\Lambda [0] = \vartheta_2^2 \vartheta_3^4 + \epsilon \vartheta_3^2 \vartheta_2^4 \quad (6.7)$$

$$\Lambda [3] = \vartheta_1^2 \vartheta_3^4 + \epsilon \vartheta_4^2 \vartheta_2^4 \quad (6.8)$$

$$\Lambda [0] = \vartheta_2^3 \vartheta_3^3 + \epsilon \vartheta_3^3 \vartheta_2^3 \quad (6.9)$$

$$\Lambda [h] = \vartheta_1^3 \vartheta_3^3 + \epsilon \vartheta_4^3 \vartheta_2^3. \quad (6.10)$$

$\epsilon_{a,b}$ and ϵ are signs satisfying the fusion constraints and $h = 1, 2$. All the other possible lattice sums vanish. For instance,

$$\Lambda [3] = \vartheta_3 \vartheta_4^3 \vartheta_1^2 \vartheta_2^2 + \vartheta_2 \vartheta_1^3 \vartheta_3^2 \vartheta_4^2 \equiv 0. \quad (6.11)$$

Performing an S modular transformation one can determine the Klein-bottle amplitude in the transverse channel

$$\tilde{\mathcal{K}} = \frac{2^2}{4 \eta^8} \sum_{a,b=0}^3 \epsilon_{b,a} \sigma_{b,a} \rho_{ab} \tilde{\Lambda} [b^a], \quad (6.12)$$

where

$$\tilde{\Lambda}_{[0]}^{[0]} = \vartheta_3^6 + \epsilon \vartheta_4^6 \tag{6.13}$$

$$\tilde{\Lambda}_{[h]}^{[0]} = \vartheta_4^3 \vartheta_3^3 + \epsilon \vartheta_3^3 \vartheta_4^3 \tag{6.14}$$

$$\tilde{\Lambda}_{[3]}^{[0]} = \vartheta_4^2 \vartheta_3^4 + \epsilon \vartheta_3^2 \vartheta_4^4 \tag{6.15}$$

$$\tilde{\Lambda}_{[0]}^{[3]} = \vartheta_2^2 \vartheta_3^4 + \epsilon \vartheta_1^2 \vartheta_4^4 \tag{6.16}$$

$$\tilde{\Lambda}_{[3]}^{[3]} = \vartheta_1^2 \vartheta_3^4 + \epsilon \vartheta_2^2 \vartheta_4^4 \tag{6.17}$$

$$\tilde{\Lambda}_{[0]}^{[h]} = \vartheta_2^3 \vartheta_3^3 + \epsilon \vartheta_1^3 \vartheta_4^3 \tag{6.18}$$

$$\tilde{\Lambda}_{[h]}^{[h]} = \vartheta_1^3 \vartheta_3^3 + \epsilon \vartheta_2^3 \vartheta_4^3 . \tag{6.19}$$

with σ_{ab} some signs given in (A.10). Choosing $\epsilon = -1$ and all the remaining signs $\epsilon_{a,b} = 1$, one finds that no massless untwisted or twisted tadpoles are present. The unoriented model is then consistent by itself and no D-branes are needed. At the massless level one finds

$$\mathcal{K}_{\text{massless}} = (V - S - C) + (2O - S - C) \tag{6.20}$$

Together with the torus contribution one is left with the minimal $\mathcal{N} = 1$ content

$$\frac{1}{2}(\mathcal{T} + \mathcal{K})_{\text{massless}} = \mathbf{G}_1 + 2 \mathbf{C}_1 . \tag{6.21}$$

6.2 Models with open strings

Here we present the simplest instance of an unoriented projection with open strings. For simplicity we consider the case of $T^6/\mathbb{Z}_{2L} \times \mathbb{Z}'_{2L} \times \mathbb{Z}_{2R} \times \mathbb{Z}'_{2R}$ with no shifts. As before, we take the T^6 at the SO(12) point. The orbifold group generators are

$$b_1 = I_{3456} , \quad b_2 = I_{1256} , \quad \bar{b}_1 = \bar{I}_{3456} , \quad \bar{b}_2 = \bar{I}_{1256} . \tag{6.22}$$

The resulting model can be written in terms of 64 characters collecting the chiral states in the a -twisted sector ($a = 0, 1, 2, 3$) with $\mathbb{Z}_{2L} \times \mathbb{Z}_{2L}$ eigenvalues (\pm, \pm) in one of the four O, V, S, C conjugacy classes of the SO(12) lattice. The complete list of characters can be found in appendix B. In particular, orbifold group invariant states in the untwisted sector are labelled by $\chi_1, \chi_5, \chi_9, \chi_{13}$. The untwisted torus is then given by

$$\mathcal{T}_{\text{unt}} = |\chi_1|^2 + |\chi_5|^2 + |\chi_9|^2 + |\chi_{13}|^2 . \tag{6.23}$$

The twisted amplitudes complete (6.23) in a modular invariant form with positive integer coefficients. We discuss the two possibilities

$$\begin{aligned} \mathcal{T}_{\mathbf{A}} = & |\chi_1 + \chi_{17} + \chi_{35} + \chi_{49}|^2 + |\chi_5 + \chi_{21} + \chi_{39} + \chi_{53}|^2 \\ & + |\chi_9 + \chi_{30} + \chi_{45} + \chi_{64}|^2 + |\chi_{13} + \chi_{26} + \chi_{41} + \chi_{60}|^2, \end{aligned} \quad (6.24)$$

$$\begin{aligned} \mathcal{T}_{\mathbf{B}} = & \chi_1 \bar{\chi}_1 + \chi_{18} \bar{\chi}_2 + \chi_{33} \bar{\chi}_3 + \chi_{52} \bar{\chi}_4 + \chi_5 \bar{\chi}_5 + \chi_{22} \bar{\chi}_6 + \chi_{37} \bar{\chi}_7 + \chi_{56} \bar{\chi}_8 + \chi_9 \bar{\chi}_9 \\ & + \chi_{29} \bar{\chi}_{10} + \chi_{47} \bar{\chi}_{11} + \chi_{61} \bar{\chi}_{12} + \chi_{13} \bar{\chi}_{13} + \chi_{25} \bar{\chi}_{14} + \chi_{43} \bar{\chi}_{15} + \chi_{57} \bar{\chi}_{16} + \chi_{17} \bar{\chi}_{17} \\ & + \chi_2 \bar{\chi}_{18} + \chi_{51} \bar{\chi}_{19} + \chi_{34} \bar{\chi}_{20} + \chi_{21} \bar{\chi}_{21} + \chi_6 \bar{\chi}_{22} + \chi_{55} \bar{\chi}_{23} + \chi_{38} \bar{\chi}_{24} + \chi_{14} \bar{\chi}_{25} \\ & + \chi_{26} \bar{\chi}_{26} + \chi_{44} \bar{\chi}_{27} + \chi_{58} \bar{\chi}_{28} + \chi_{10} \bar{\chi}_{29} + \chi_{30} \bar{\chi}_{30} + \chi_{48} \bar{\chi}_{31} + \chi_{62} \bar{\chi}_{32} + \chi_3 \bar{\chi}_{33} \\ & + \chi_{20} \bar{\chi}_{34} + \chi_{35} \bar{\chi}_{35} + \chi_{50} \bar{\chi}_{36} + \chi_7 \bar{\chi}_{37} + \chi_{24} \bar{\chi}_{38} + \chi_{39} \bar{\chi}_{39} + \chi_{54} \bar{\chi}_{40} + \chi_{41} \bar{\chi}_{41} \\ & + \chi_{59} \bar{\chi}_{42} + \chi_{15} \bar{\chi}_{43} + \chi_{27} \bar{\chi}_{44} + \chi_{45} \bar{\chi}_{45} + \chi_{63} \bar{\chi}_{46} + \chi_{11} \bar{\chi}_{47} + \chi_{31} \bar{\chi}_{48} + \chi_{49} \bar{\chi}_{49} \\ & + \chi_{36} \bar{\chi}_{50} + \chi_{19} \bar{\chi}_{51} + \chi_4 \bar{\chi}_{52} + \chi_{53} \bar{\chi}_{53} + \chi_{40} \bar{\chi}_{54} + \chi_{23} \bar{\chi}_{55} + \chi_8 \bar{\chi}_{56} + \chi_{16} \bar{\chi}_{57} \\ & + \chi_{28} \bar{\chi}_{58} + \chi_{42} \bar{\chi}_{59} + \chi_{60} \bar{\chi}_{60} + \chi_{12} \bar{\chi}_{61} + \chi_{32} \bar{\chi}_{62} + \chi_{46} \bar{\chi}_{63} + \chi_{64} \bar{\chi}_{64}. \end{aligned} \quad (6.25)$$

They coincide in the untwisted sector and are distinguished by the pairing of states in the twisted sectors, namely they correspond to different choices of discrete torsion giving rise to different modular invariants [8]. In particular, case **A** corresponds to a modular invariant with extended symmetry that gives back the toroidal compactification of Type IIB on the T^6 based on the lattice of $SO(12)$. On the other hand, case **B** corresponds to a permutation modular invariant with effective Hodge numbers (15, 15). Indeed, out of the 64 original characters, the set of massless characters consists in

$$\{\chi_1, \chi_2, \chi_3, \chi_4, \chi_{17}, \chi_{18}, \chi_{23}, \chi_{24}, \chi_{33}, \chi_{35}, \chi_{38}, \chi_{40}, \chi_{49}, \chi_{52}, \chi_{54}, \chi_{55}\}, \quad (6.26)$$

with $\chi_1 = V - S - C + \dots$ and $\chi_i = 2O - S - C + \dots$ for the remaining ones. Plugging the above expansions into the expressions for the two torus amplitudes one finds

$$\begin{aligned} (\mathcal{T}_{\mathbf{A}})_{\text{massless}} &= |V + 6O - 4S - 4C|^2, \\ (\mathcal{T}_{\mathbf{B}})_{\text{massless}} &= |V - S - C|^2 + 15|2O - S - C|^2. \end{aligned} \quad (6.27)$$

The Klein-bottle amplitude follows from \mathcal{T}_A and \mathcal{T}_B by reducing to their diagonal components. In both cases one finds

$$\begin{aligned} \mathcal{K} = & \chi_1 + \chi_{17} + \chi_{35} + \chi_{49} + \chi_5 + \chi_{21} + \chi_{39} + \chi_{53} \\ & + \chi_9 + \chi_{30} + \chi_{45} + \chi_{64} + \chi_{13} + \chi_{26} + \chi_{41} + \chi_{60}, \end{aligned} \quad (6.28)$$

that produces

$$\mathcal{K} = (V - S - C) + 3(2O - S - C) \quad (6.29)$$

at the massless level. The unoriented projection results in case **A** into the supegravity multiplet with 6 vector multiplets of $\mathcal{N} = 4$, while in case **B** it leads to $\mathcal{N} = 1$ supergravity with 6 vector multiplets and 25 chiral multiplets.

Going to the transverse channel one finds

$$\tilde{\mathcal{K}} = 2^3(\chi_1 + \chi_{17} + \chi_{35} + \chi_{49}) . \quad (6.30)$$

The tadpoles can be cancelled by adding the transverse Annulus and Moebius amplitudes

$$\begin{aligned} \tilde{\mathcal{A}} = & 2^{-3}(\chi_1 + \chi_{17} + \chi_{35} + \chi_{49})(n_1 + n_2 + \bar{n}_1 + \bar{n}_2)^2 \\ & + 2^{-3}(\chi_5 + \chi_{21} + \chi_{39} + \chi_{53})(n_1 - n_2 + \bar{n}_1 - \bar{n}_2)^2 \\ & + 2^{-3}(\chi_9 + \chi_{30} + \chi_{45} + \chi_{64})(n_1 + n_2 - \bar{n}_1 - \bar{n}_2)^2 \\ & + 2^{-3}(\chi_{13} + \chi_{26} + \chi_{41} + \chi_{60})(n_1 - n_2 - \bar{n}_1 + \bar{n}_2)^2, \end{aligned} \quad (6.31)$$

$$\tilde{\mathcal{M}} = -(\chi_1 + \chi_{17} + \chi_{35} + \chi_{49})(n_1 + n_2 + \bar{n}_1 + \bar{n}_2), \quad (6.32)$$

provided

$$n_1 + n_2 = 4 . \quad (6.33)$$

Finally, applying S and $P = T^{\frac{1}{2}}ST^2ST^{\frac{1}{2}}$ modular transformations one finds the direct amplitudes

$$\begin{aligned} \mathcal{A} = & (\chi_1 + \chi_{17} + \chi_{35} + \chi_{49})(2n_1\bar{n}_1 + 2n_2\bar{n}_2) \\ & + (\chi_5 + \chi_{21} + \chi_{39} + \chi_{53})(n_1^2 + n_2^2 + \bar{n}_1^2 + \bar{n}_2^2) \\ & + (\chi_9 + \chi_{30} + \chi_{45} + \chi_{64})(2n_1n_2 + 2\bar{n}_1\bar{n}_2) \\ & + (\chi_{13} + \chi_{26} + \chi_{41} + \chi_{60})(2n_1\bar{n}_2 + 2n_2\bar{n}_1), \end{aligned} \quad (6.34)$$

$$\mathcal{M} = (\chi_5 + \chi_{21} + \chi_{39} + \chi_{53})(n_1 + n_2 + \bar{n}_1 + \bar{n}_2) . \quad (6.35)$$

The massless open string spectrum, encoded in $(\mathcal{A} + \mathcal{M})/2$, is that of $\mathcal{N} = 4$ SYM with gauge group $U(N) \times U(4 - N)$. Notice that in case **B** only an $\mathcal{N} = 1$ fraction of the $\mathcal{N} = 4$ brane supersymmetry is preserved by the bulk theory. An analogous behavior can be observed in other cases, most notably the open descendants of the D_{odd} series of $SU(2)$ WZW models [30, 31].

7 Conclusions and perspectives

In perturbative string theory, moduli fields are exactly marginal deformations of the underlying conformal field theory. In the low energy description, they correspond to perturbatively exact flat directions of the scalar potential. In the present paper, we have exploited \mathbb{Z}_2 chiral twists and shifts in the search of calculable Type IIB models with few moduli. We have explored both Left-Right symmetric, though non-geometric, models with $\mathcal{N} = 1_L + 1_R$ spacetime susy and Left-Right asymmetric models with $\mathcal{N} = 1_L + 0_R$ spacetime susy. We have found a finite series of models enjoying $\mathcal{N} = 1_L + 1_R$ spacetime susy with very low “effective” Hodge numbers (h_{11}, h_{21}) given by

$$\begin{aligned} & (n, n) & n = 1, 2, 3, 4, 5, 9 \\ & (2n, 2n + 6), (2n + 6, 2n) & n = 0, 1, 2 \\ & (2n + 3, 2n + 15), (2n + 15, 2n + 3) & n = 0, 1 \end{aligned} \quad (7.1)$$

Most of these models have no counterpart in previous CY or RCFT scans [20, 21]. We have studied the “minimal” model with $h_{11} = h_{21} = 1$ in details and constructed one of its $\mathcal{N} = 1$ unoriented descendants with no open strings. This model exhibits the minimal (as far as we know) $\mathcal{N} = 1$ field content found so far in the moduli space of perturbative string compactifications. We cannot exclude the possibility that more general chiral twists and shifts could give rise to perturbative Type IIB models with $\mathcal{N} = 1$ spacetime susy and only the universal dilaton chiral multiplet or to an $\mathcal{N} = 2$ model with $h_{11} = h_{21} = 0$.⁵

Our main motivation was to identify convenient starting points for calculable orientifold constructions exhibiting complete moduli stabilization. We find that asymmetric twists and shifts can be easily combined in order to freeze out most closed string moduli. The effect on open string moduli is subtler. The only model with open unoriented strings, we have analyzed in some detail, enjoys extended $\mathcal{N} = 4$ susy in the open sector and is thus non-chiral. Apparently there is some tension between chirality and moduli stabilization.⁶ The interesting question of whether D-branes with phenomenologically viable gauge group and chiral matter contents can be accommodated in this picture remains open.

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A Some definitions

In this appendix we collect some useful formulas illustrating our conventions. We adopt the following definition for the Jacobi theta functions:

$$\vartheta_{[b]}^{[a]}(v|\tau) = \sum_{n \in \mathbb{Z}} q^{\frac{1}{2}(n+a)^2} e^{2\pi i(n+a)(v+b)} . \tag{A.1}$$

⁵A Left-Right asymmetric “minimal” model with $\mathcal{N} = 2_L + 0_R$ spacetime susy and only the dilaton vector (!) multiplet has been constructed by similar means in [24] but it does not admit an obvious unoriented projection.

⁶P. Camara and others share our viewpoint.

The Characters of $SO(2n)$ level one are

$$\begin{aligned} O_{2n} &= \frac{1}{2\eta^n} (\vartheta_3^n + \vartheta_4^n) \quad ; & V_{2n} &= \frac{1}{2\eta^n} (\vartheta_3^n - \vartheta_4^n) \quad ; \\ S_{2n} &= \frac{1}{2\eta^n} (\vartheta_2^n + i^{-n}\vartheta_1^n) \quad ; & C_{2n} &= \frac{1}{2\eta^n} (\vartheta_2^n - i^{-n}\vartheta_1^n), \end{aligned} \quad (A.2)$$

and the corresponding ground states can be described as

$$\begin{aligned} q^{\frac{1}{6}} O_4 &= (1, 1) + \dots , \\ q^{\frac{1}{6}} V_4 &= (2, 2) q^{\frac{1}{2}} + \dots , \\ q^{\frac{1}{6}} S_4 &= (1, 2) q^{\frac{1}{4}} + \dots , \\ q^{\frac{1}{6}} C_4 &= (2, 1) q^{\frac{1}{4}} + \dots . \end{aligned} \quad (A.3)$$

The modular transformation matrices on the characters of $SO(2n)$ level one are the following

$$T = e^{-\frac{i\pi n}{12}} \text{Diag} (1, -1, e^{\frac{i\pi n}{4}}, e^{\frac{i\pi n}{4}}), \quad (A.4)$$

$$S = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix}, \quad (A.5)$$

$$P = \begin{pmatrix} c & s & 0 & 0 \\ s & -c & 0 & 0 \\ 0 & 0 & c\xi & is\xi \\ 0 & 0 & is\xi & c\xi \end{pmatrix}, \quad (A.6)$$

with $s = \sin \frac{n\pi}{4}$, $c = \cos \frac{n\pi}{4}$ and $\xi = e^{-\frac{in\pi}{4}}$.

The space-time characters for the supersymmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ model are [32]

$$\begin{aligned} \tau_{00} &= V_2 O_2 O_2 O_2 + O_2 V_2 V_2 V_2 - S_2 S_2 S_2 S_2 - C_2 C_2 C_2 C_2 \\ \tau_{01} &= O_2 V_2 O_2 O_2 + V_2 O_2 V_2 V_2 - C_2 C_2 S_2 S_2 - S_2 S_2 C_2 C_2 \\ \tau_{02} &= O_2 O_2 V_2 O_2 + V_2 V_2 O_2 V_2 - C_2 S_2 C_2 S_2 - S_2 C_2 S_2 C_2 \\ \tau_{03} &= O_2 O_2 O_2 V_2 + V_2 V_2 V_2 O_2 - C_2 S_2 S_2 C_2 - S_2 C_2 C_2 S_2 \\ \tau_{10} &= V_2 O_2 S_2 C_2 + O_2 V_2 C_2 S_2 - S_2 S_2 V_2 O_2 - C_2 C_2 O_2 V_2 \\ \tau_{11} &= O_2 V_2 S_2 C_2 + V_2 O_2 C_2 S_2 - S_2 S_2 O_2 V_2 - C_2 C_2 V_2 O_2 \\ \tau_{12} &= O_2 O_2 C_2 C_2 + V_2 V_2 S_2 S_2 - S_2 C_2 V_2 V_2 - C_2 S_2 O_2 O_2 \\ \tau_{13} &= O_2 O_2 S_2 S_2 + V_2 V_2 C_2 C_2 - C_2 S_2 V_2 V_2 - S_2 C_2 O_2 O_2 \\ \tau_{20} &= V_2 S_2 O_2 C_2 + O_2 C_2 V_2 S_2 - S_2 V_2 S_2 O_2 - C_2 O_2 C_2 V_2 \\ \tau_{21} &= O_2 C_2 O_2 C_2 + V_2 S_2 V_2 S_2 - C_2 O_2 S_2 O_2 - S_2 V_2 C_2 V_2 \\ \tau_{22} &= O_2 S_2 V_2 C_2 + V_2 C_2 O_2 S_2 - C_2 V_2 C_2 O_2 - S_2 O_2 S_2 V_2 \\ \tau_{23} &= O_2 S_2 O_2 S_2 + V_2 C_2 V_2 C_2 - C_2 V_2 S_2 V_2 - S_2 O_2 C_2 O_2 \end{aligned}$$

$$\begin{aligned}
\tau_{30} &= V_2 S_2 C_2 O_2 + O_2 C_2 S_2 V_2 - C_2 O_2 V_2 C_2 - S_2 V_2 O_2 S_2 \\
\tau_{31} &= O_2 C_2 C_2 O_2 + V_2 S_2 S_2 V_2 - C_2 O_2 O_2 S_2 - S_2 V_2 V_2 C_2 \\
\tau_{32} &= O_2 S_2 S_2 O_2 + V_2 C_2 C_2 V_2 - C_2 V_2 V_2 S_2 - S_2 O_2 O_2 C_2 \\
\tau_{33} &= O_2 S_2 C_2 V_2 + V_2 C_2 S_2 O_2 - S_2 O_2 V_2 S_2 - C_2 V_2 O_2 C_2,
\end{aligned} \tag{A.7}$$

while the corresponding amplitudes $\rho_{a,b}$ expressed in terms of the previous characters result:

$$\begin{aligned}
\rho_{a0} &= \tau_{a0} + \tau_{a1} + \tau_{a2} + \tau_{a3}, & \rho_{a1} &= \tau_{a0} + \tau_{a1} - \tau_{a2} - \tau_{a3}, \\
\rho_{a2} &= \tau_{a0} - \tau_{a1} + \tau_{a2} - \tau_{a3}, & \rho_{a3} &= \tau_{a0} - \tau_{a1} - \tau_{a2} + \tau_{a3}.
\end{aligned} \tag{A.8}$$

It is useful to recall their S -modular transformations

$$\rho_{ab}(-1/\tau) = \sigma_{ab} \rho_{ba}(\tau), \tag{A.9}$$

where the phases are

$$\begin{aligned}
\sigma_{00} = \sigma_{01} = \sigma_{02} = \sigma_{03} = \sigma_{10} = \sigma_{20} = \sigma_{30} &= 1, \\
\sigma_{11} = \sigma_{22} = \sigma_{33} &= -1, \\
\sigma_{13} = -\sigma_{12} = -\sigma_{21} = -\sigma_{23} = \sigma_{31} = -\sigma_{32} &= i.
\end{aligned} \tag{A.10}$$

B Characters of $T^6/\mathbb{Z}_{2L} \times \mathbb{Z}'_{2L} \times \mathbb{Z}_{2R} \times \mathbb{Z}'_{2R}$

In this appendix we list the 64 characters corresponding to the chiral amplitudes of the Type IIB compactification on $T^6/\mathbb{Z}_{2L} \times \mathbb{Z}'_{2L} \times \mathbb{Z}_{2R} \times \mathbb{Z}'_{2R}$ that enter the partition functions of the models discussed in section 6.

$$\begin{aligned}
\chi_1 &= (O_2 O_2 O_2 O_6 + V_2 V_2 V_2 V_6) \tau_{00} + (O_2 V_2 V_2 O_6 + V_2 O_2 O_2 V_6) \tau_{01} + (V_2 O_2 V_2 O_6 + O_2 V_2 O_2 V_6) \tau_{02} + (V_2 V_2 O_2 O_6 + O_2 O_2 V_2 V_6) \tau_{03} \\
\chi_2 &= (O_2 V_2 V_2 O_6 + V_2 O_2 O_2 V_6) \tau_{00} + (O_2 O_2 O_2 O_6 + V_2 V_2 V_2 V_6) \tau_{01} + (V_2 V_2 O_2 O_6 + O_2 O_2 V_2 V_6) \tau_{02} + (V_2 O_2 V_2 O_6 + O_2 V_2 O_2 V_6) \tau_{03} \\
\chi_3 &= (V_2 O_2 V_2 O_6 + O_2 V_2 O_2 V_6) \tau_{00} + (V_2 V_2 O_2 O_6 + O_2 O_2 V_2 V_6) \tau_{01} + (O_2 O_2 O_2 O_6 + V_2 V_2 V_2 V_6) \tau_{02} + (O_2 V_2 V_2 O_6 + V_2 O_2 O_2 V_6) \tau_{03} \\
\chi_4 &= (V_2 V_2 O_2 O_6 + O_2 O_2 V_2 V_6) \tau_{00} + (V_2 O_2 V_2 O_6 + O_2 V_2 O_2 V_6) \tau_{01} + (O_2 V_2 V_2 O_6 + V_2 O_2 O_2 V_6) \tau_{02} + (O_2 O_2 O_2 O_6 + V_2 V_2 V_2 V_6) \tau_{03} \\
\chi_5 &= (V_2 V_2 V_2 O_6 + O_2 O_2 O_2 V_6) \tau_{00} + (V_2 O_2 O_2 O_6 + O_2 V_2 V_2 V_6) \tau_{01} + (O_2 V_2 O_2 O_6 + V_2 O_2 O_2 V_6) \tau_{02} + (O_2 O_2 V_2 O_6 + V_2 V_2 O_2 V_6) \tau_{03} \\
\chi_6 &= (V_2 O_2 O_2 O_6 + O_2 V_2 V_2 V_6) \tau_{00} + (V_2 V_2 V_2 O_6 + O_2 O_2 O_2 V_6) \tau_{01} + (O_2 O_2 V_2 O_6 + V_2 V_2 O_2 V_6) \tau_{02} + (O_2 V_2 O_2 O_6 + V_2 O_2 V_2 V_6) \tau_{03} \\
\chi_7 &= (O_2 V_2 O_2 O_6 + V_2 O_2 V_2 V_6) \tau_{00} + (O_2 O_2 V_2 O_6 + V_2 V_2 O_2 V_6) \tau_{01} + (V_2 V_2 V_2 O_6 + O_2 O_2 O_2 V_6) \tau_{02} + (V_2 O_2 O_2 O_6 + O_2 V_2 V_2 V_6) \tau_{03} \\
\chi_8 &= (O_2 O_2 V_2 O_6 + V_2 V_2 O_2 V_6) \tau_{00} + (O_2 V_2 O_2 O_6 + V_2 O_2 V_2 V_6) \tau_{01} + (V_2 O_2 O_2 O_6 + O_2 V_2 V_2 V_6) \tau_{02} + (V_2 V_2 V_2 O_6 + O_2 O_2 O_2 V_6) \tau_{03} \\
\chi_9 &= (C_2 C_2 C_2 C_6 + S_2 S_2 S_2 S_6) \tau_{00} + (C_2 S_2 S_2 C_6 + S_2 C_2 C_2 S_6) \tau_{01} + (S_2 C_2 S_2 C_6 + C_2 S_2 C_2 S_6) \tau_{02} + (S_2 S_2 C_2 C_6 + C_2 C_2 S_2 S_6) \tau_{03} \\
\chi_{10} &= (C_2 S_2 S_2 C_6 + S_2 C_2 C_2 S_6) \tau_{00} + (C_2 C_2 C_2 C_6 + S_2 S_2 S_2 S_6) \tau_{01} + (S_2 S_2 C_2 C_6 + C_2 C_2 S_2 S_6) \tau_{02} + (S_2 C_2 S_2 C_6 + C_2 S_2 C_2 S_6) \tau_{03} \\
\chi_{11} &= (S_2 C_2 S_2 C_6 + C_2 S_2 C_2 S_6) \tau_{00} + (S_2 S_2 C_2 C_6 + C_2 C_2 S_2 S_6) \tau_{01} + (C_2 C_2 C_2 C_6 + S_2 S_2 S_2 S_6) \tau_{02} + (C_2 S_2 S_2 C_6 + S_2 C_2 C_2 S_6) \tau_{03} \\
\chi_{12} &= (S_2 S_2 C_2 C_6 + C_2 C_2 S_2 S_6) \tau_{00} + (S_2 C_2 S_2 C_6 + C_2 S_2 C_2 S_6) \tau_{01} + (C_2 S_2 S_2 C_6 + S_2 C_2 C_2 S_6) \tau_{02} + (C_2 C_2 C_2 C_6 + S_2 S_2 S_2 S_6) \tau_{03} \\
\chi_{13} &= (S_2 S_2 S_2 C_6 + C_2 C_2 C_2 S_6) \tau_{00} + (S_2 C_2 C_2 C_6 + C_2 S_2 S_2 S_6) \tau_{01} + (C_2 S_2 C_2 C_6 + S_2 C_2 S_2 S_6) \tau_{02} + (C_2 C_2 S_2 C_6 + S_2 S_2 C_2 S_6) \tau_{03} \\
\chi_{14} &= (S_2 C_2 C_2 C_6 + C_2 S_2 S_2 S_6) \tau_{00} + (S_2 S_2 S_2 C_6 + C_2 C_2 C_2 S_6) \tau_{01} + (C_2 C_2 S_2 C_6 + S_2 S_2 C_2 S_6) \tau_{02} + (C_2 S_2 C_2 C_6 + S_2 C_2 S_2 S_6) \tau_{03} \\
\chi_{15} &= (C_2 S_2 C_2 C_6 + S_2 C_2 S_2 S_6) \tau_{00} + (C_2 C_2 S_2 C_6 + S_2 S_2 C_2 S_6) \tau_{01} + (S_2 S_2 S_2 C_6 + C_2 C_2 C_2 S_6) \tau_{02} + (S_2 C_2 C_2 C_6 + C_2 S_2 S_2 S_6) \tau_{03} \\
\chi_{16} &= (C_2 C_2 S_2 C_6 + S_2 S_2 C_2 S_6) \tau_{00} + (C_2 S_2 C_2 C_6 + S_2 C_2 S_2 S_6) \tau_{01} + (S_2 C_2 C_2 C_6 + C_2 S_2 S_2 S_6) \tau_{02} + (S_2 S_2 S_2 C_6 + C_2 C_2 C_2 S_6) \tau_{03} \\
\chi_{17} &= (V_2 S_2 S_2 O_6 + O_2 C_2 C_2 V_6) \tau_{10} + (V_2 C_2 C_2 O_6 + O_2 S_2 S_2 V_6) \tau_{11} + (O_2 S_2 C_2 O_6 + V_2 C_2 S_2 V_6) \tau_{12} + (O_2 C_2 S_2 O_6 + V_2 S_2 C_2 V_6) \tau_{13} \\
\chi_{18} &= (V_2 C_2 C_2 O_6 + O_2 S_2 S_2 V_6) \tau_{10} + (V_2 S_2 S_2 O_6 + O_2 C_2 C_2 V_6) \tau_{11} + (O_2 C_2 S_2 O_6 + V_2 S_2 C_2 V_6) \tau_{12} + (O_2 S_2 C_2 O_6 + V_2 C_2 S_2 V_6) \tau_{13} \\
\chi_{19} &= (O_2 S_2 C_2 O_6 + V_2 C_2 S_2 V_6) \tau_{10} + (O_2 C_2 S_2 O_6 + V_2 S_2 C_2 V_6) \tau_{11} + (V_2 S_2 S_2 O_6 + O_2 C_2 C_2 V_6) \tau_{12} + (V_2 C_2 C_2 O_6 + O_2 S_2 S_2 V_6) \tau_{13} \\
\chi_{20} &= (O_2 C_2 S_2 O_6 + V_2 S_2 C_2 V_6) \tau_{10} + (O_2 S_2 C_2 O_6 + V_2 C_2 S_2 V_6) \tau_{11} + (V_2 C_2 C_2 O_6 + O_2 S_2 S_2 V_6) \tau_{12} + (V_2 S_2 S_2 O_6 + O_2 C_2 C_2 V_6) \tau_{13} \\
\chi_{21} &= (O_2 C_2 C_2 O_6 + V_2 S_2 S_2 V_6) \tau_{10} + (O_2 S_2 S_2 O_6 + V_2 C_2 C_2 V_6) \tau_{11} + (V_2 C_2 S_2 O_6 + O_2 S_2 C_2 V_6) \tau_{12} + (V_2 S_2 C_2 O_6 + O_2 C_2 S_2 V_6) \tau_{13} \\
\chi_{22} &= (O_2 S_2 S_2 O_6 + V_2 C_2 C_2 V_6) \tau_{10} + (O_2 C_2 C_2 O_6 + V_2 S_2 S_2 V_6) \tau_{11} + (V_2 S_2 C_2 O_6 + O_2 C_2 S_2 V_6) \tau_{12} + (V_2 C_2 S_2 O_6 + O_2 S_2 C_2 V_6) \tau_{13} \\
\chi_{23} &= (V_2 C_2 S_2 O_6 + O_2 S_2 C_2 V_6) \tau_{10} + (V_2 S_2 C_2 O_6 + O_2 C_2 S_2 V_6) \tau_{11} + (O_2 C_2 C_2 O_6 + V_2 S_2 S_2 V_6) \tau_{12} + (O_2 S_2 S_2 O_6 + V_2 C_2 C_2 V_6) \tau_{13} \\
\chi_{24} &= (V_2 S_2 C_2 O_6 + O_2 C_2 S_2 V_6) \tau_{10} + (V_2 C_2 S_2 O_6 + O_2 S_2 C_2 V_6) \tau_{11} + (O_2 S_2 S_2 O_6 + V_2 C_2 C_2 V_6) \tau_{12} + (O_2 C_2 C_2 O_6 + V_2 S_2 S_2 V_6) \tau_{13} \\
\chi_{25} &= (C_2 O_2 O_2 C_6 + S_2 V_2 V_2 S_6) \tau_{10} + (C_2 V_2 V_2 C_6 + S_2 O_2 O_2 S_6) \tau_{11} + (S_2 O_2 V_2 C_6 + C_2 V_2 O_2 S_6) \tau_{12} + (S_2 V_2 O_2 C_6 + C_2 O_2 V_2 S_6) \tau_{13}
\end{aligned}$$

$$\begin{aligned}
\chi_{26} &= (C_2 V_2 V_2 C_6 + S_2 O_2 O_2 S_6) \tau_{10} + (C_2 O_2 O_2 C_6 + S_2 V_2 V_2 S_6) \tau_{11} + (S_2 V_2 O_2 C_6 + C_2 O_2 V_2 S_6) \tau_{12} + (S_2 O_2 V_2 C_6 + C_2 V_2 O_2 S_6) \tau_{13} \\
\chi_{27} &= (S_2 O_2 V_2 C_6 + C_2 V_2 O_2 S_6) \tau_{10} + (S_2 V_2 O_2 C_6 + C_2 O_2 V_2 S_6) \tau_{11} + (C_2 O_2 O_2 C_6 + S_2 V_2 V_2 S_6) \tau_{12} + (C_2 V_2 V_2 C_6 + S_2 O_2 O_2 S_6) \tau_{13} \\
\chi_{28} &= (S_2 V_2 O_2 C_6 + C_2 O_2 V_2 S_6) \tau_{10} + (S_2 O_2 V_2 C_6 + C_2 V_2 O_2 S_6) \tau_{11} + (C_2 V_2 V_2 C_6 + S_2 O_2 O_2 S_6) \tau_{12} + (C_2 O_2 O_2 C_6 + S_2 V_2 V_2 S_6) \tau_{13} \\
\chi_{29} &= (S_2 V_2 V_2 C_6 + C_2 O_2 O_2 S_6) \tau_{10} + (S_2 O_2 O_2 C_6 + C_2 V_2 V_2 S_6) \tau_{11} + (C_2 V_2 O_2 C_6 + S_2 O_2 V_2 S_6) \tau_{12} + (C_2 O_2 V_2 C_6 + S_2 V_2 O_2 S_6) \tau_{13} \\
\chi_{30} &= (S_2 O_2 O_2 C_6 + C_2 V_2 V_2 S_6) \tau_{10} + (S_2 V_2 V_2 C_6 + C_2 O_2 O_2 S_6) \tau_{11} + (C_2 O_2 V_2 C_6 + S_2 V_2 O_2 S_6) \tau_{12} + (C_2 V_2 O_2 C_6 + S_2 O_2 V_2 S_6) \tau_{13} \\
\chi_{31} &= (C_2 V_2 O_2 C_6 + S_2 O_2 V_2 S_6) \tau_{10} + (C_2 O_2 V_2 C_6 + S_2 V_2 O_2 S_6) \tau_{11} + (S_2 V_2 V_2 C_6 + C_2 O_2 O_2 S_6) \tau_{12} + (S_2 O_2 O_2 C_6 + C_2 V_2 V_2 S_6) \tau_{13} \\
\chi_{32} &= (C_2 O_2 V_2 C_6 + S_2 V_2 O_2 S_6) \tau_{10} + (C_2 V_2 O_2 C_6 + S_2 O_2 V_2 S_6) \tau_{11} + (S_2 O_2 O_2 C_6 + C_2 V_2 V_2 S_6) \tau_{12} + (S_2 V_2 V_2 C_6 + C_2 O_2 O_2 S_6) \tau_{13} \\
\chi_{33} &= (S_2 V_2 S_2 O_6 + C_2 O_2 C_2 V_6) \tau_{20} + (S_2 O_2 C_2 O_6 + C_2 V_2 S_2 V_6) \tau_{21} + (C_2 V_2 C_2 O_6 + S_2 O_2 S_2 V_6) \tau_{22} + (C_2 O_2 S_2 O_6 + S_2 V_2 C_2 V_6) \tau_{23} \\
\chi_{34} &= (S_2 O_2 C_2 O_6 + C_2 V_2 S_2 V_6) \tau_{20} + (S_2 V_2 S_2 O_6 + C_2 O_2 C_2 V_6) \tau_{21} + (C_2 O_2 S_2 O_6 + S_2 V_2 C_2 V_6) \tau_{22} + (C_2 V_2 C_2 O_6 + S_2 O_2 S_2 V_6) \tau_{23} \\
\chi_{35} &= (C_2 V_2 C_2 O_6 + S_2 O_2 S_2 V_6) \tau_{20} + (C_2 O_2 S_2 O_6 + S_2 V_2 C_2 V_6) \tau_{21} + (S_2 V_2 S_2 O_6 + C_2 O_2 C_2 V_6) \tau_{22} + (S_2 O_2 C_2 O_6 + C_2 V_2 S_2 V_6) \tau_{23} \\
\chi_{36} &= (C_2 O_2 S_2 O_6 + S_2 V_2 C_2 V_6) \tau_{20} + (C_2 V_2 C_2 O_6 + S_2 O_2 S_2 V_6) \tau_{21} + (S_2 O_2 C_2 O_6 + C_2 V_2 S_2 V_6) \tau_{22} + (S_2 V_2 S_2 O_6 + C_2 O_2 C_2 V_6) \tau_{23} \\
\chi_{37} &= (C_2 O_2 C_2 O_6 + S_2 V_2 S_2 V_6) \tau_{20} + (C_2 V_2 S_2 O_6 + S_2 O_2 C_2 V_6) \tau_{21} + (S_2 O_2 S_2 O_6 + C_2 V_2 C_2 V_6) \tau_{22} + (S_2 V_2 C_2 O_6 + C_2 O_2 S_2 V_6) \tau_{23} \\
\chi_{38} &= (C_2 V_2 S_2 O_6 + S_2 O_2 C_2 V_6) \tau_{20} + (C_2 O_2 C_2 O_6 + S_2 V_2 S_2 V_6) \tau_{21} + (S_2 V_2 C_2 O_6 + C_2 O_2 S_2 V_6) \tau_{22} + (S_2 O_2 S_2 O_6 + C_2 V_2 C_2 V_6) \tau_{23} \\
\chi_{39} &= (S_2 O_2 S_2 O_6 + C_2 V_2 C_2 V_6) \tau_{20} + (S_2 V_2 C_2 O_6 + C_2 O_2 S_2 V_6) \tau_{21} + (C_2 O_2 C_2 O_6 + S_2 V_2 S_2 V_6) \tau_{22} + (C_2 V_2 S_2 O_6 + S_2 O_2 C_2 V_6) \tau_{23} \\
\chi_{40} &= (S_2 V_2 C_2 O_6 + C_2 O_2 S_2 V_6) \tau_{20} + (S_2 O_2 S_2 O_6 + C_2 V_2 C_2 V_6) \tau_{21} + (C_2 V_2 S_2 O_6 + S_2 O_2 C_2 V_6) \tau_{22} + (C_2 O_2 C_2 O_6 + S_2 V_2 S_2 V_6) \tau_{23} \\
\chi_{41} &= (O_2 C_2 O_2 C_6 + V_2 S_2 V_2 S_6) \tau_{20} + (O_2 S_2 V_2 C_6 + V_2 C_2 O_2 S_6) \tau_{21} + (V_2 C_2 V_2 C_6 + O_2 S_2 O_2 S_6) \tau_{22} + (V_2 S_2 O_2 C_6 + O_2 C_2 V_2 S_6) \tau_{23} \\
\chi_{42} &= (O_2 S_2 V_2 C_6 + V_2 C_2 O_2 S_6) \tau_{20} + (O_2 C_2 O_2 C_6 + V_2 S_2 V_2 S_6) \tau_{21} + (V_2 S_2 O_2 C_6 + O_2 C_2 V_2 S_6) \tau_{22} + (V_2 C_2 V_2 C_6 + O_2 S_2 O_2 S_6) \tau_{23} \\
\chi_{43} &= (V_2 C_2 V_2 C_6 + O_2 S_2 V_2 S_6) \tau_{20} + (V_2 S_2 O_2 C_6 + O_2 C_2 V_2 S_6) \tau_{21} + (O_2 C_2 O_2 C_6 + V_2 S_2 V_2 S_6) \tau_{22} + (O_2 S_2 V_2 C_6 + V_2 C_2 O_2 S_6) \tau_{23} \\
\chi_{44} &= (V_2 S_2 O_2 C_6 + O_2 C_2 V_2 S_6) \tau_{20} + (V_2 C_2 V_2 C_6 + O_2 S_2 V_2 S_6) \tau_{21} + (O_2 S_2 V_2 C_6 + V_2 C_2 O_2 S_6) \tau_{22} + (O_2 C_2 O_2 C_6 + V_2 S_2 V_2 S_6) \tau_{23} \\
\chi_{45} &= (V_2 S_2 V_2 C_6 + O_2 C_2 O_2 S_6) \tau_{20} + (V_2 C_2 O_2 C_6 + O_2 S_2 V_2 S_6) \tau_{21} + (O_2 S_2 O_2 C_6 + V_2 C_2 V_2 S_6) \tau_{22} + (O_2 C_2 V_2 C_6 + V_2 S_2 O_2 S_6) \tau_{23} \\
\chi_{46} &= (V_2 C_2 O_2 C_6 + O_2 S_2 V_2 S_6) \tau_{20} + (V_2 S_2 V_2 C_6 + O_2 C_2 O_2 S_6) \tau_{21} + (O_2 C_2 V_2 C_6 + V_2 S_2 O_2 S_6) \tau_{22} + (O_2 S_2 O_2 C_6 + V_2 C_2 V_2 S_6) \tau_{23} \\
\chi_{47} &= (O_2 S_2 O_2 C_6 + V_2 C_2 V_2 S_6) \tau_{20} + (O_2 C_2 V_2 C_6 + V_2 S_2 O_2 S_6) \tau_{21} + (V_2 S_2 V_2 C_6 + O_2 C_2 O_2 S_6) \tau_{22} + (V_2 C_2 O_2 C_6 + O_2 S_2 V_2 S_6) \tau_{23} \\
\chi_{48} &= (O_2 C_2 V_2 C_6 + V_2 S_2 O_2 S_6) \tau_{20} + (O_2 S_2 O_2 C_6 + V_2 C_2 V_2 S_6) \tau_{21} + (V_2 C_2 O_2 C_6 + O_2 S_2 V_2 S_6) \tau_{22} + (V_2 S_2 V_2 C_6 + O_2 C_2 O_2 S_6) \tau_{23} \\
\chi_{49} &= (S_2 S_2 V_2 O_6 + C_2 C_2 O_2 V_6) \tau_{30} + (S_2 C_2 O_2 O_6 + C_2 S_2 V_2 V_6) \tau_{31} + (C_2 S_2 O_2 O_6 + S_2 C_2 V_2 V_6) \tau_{32} + (C_2 C_2 V_2 O_6 + S_2 S_2 O_2 V_6) \tau_{33} \\
\chi_{50} &= (S_2 C_2 O_2 O_6 + C_2 S_2 V_2 V_6) \tau_{30} + (S_2 S_2 V_2 O_6 + C_2 C_2 O_2 V_6) \tau_{31} + (C_2 C_2 V_2 O_6 + S_2 S_2 O_2 V_6) \tau_{32} + (C_2 S_2 O_2 O_6 + S_2 C_2 V_2 V_6) \tau_{33} \\
\chi_{51} &= (C_2 S_2 O_2 O_6 + S_2 C_2 V_2 V_6) \tau_{30} + (C_2 C_2 V_2 O_6 + S_2 S_2 O_2 V_6) \tau_{31} + (S_2 S_2 V_2 O_6 + C_2 C_2 O_2 V_6) \tau_{32} + (S_2 C_2 O_2 O_6 + C_2 S_2 V_2 V_6) \tau_{33} \\
\chi_{52} &= (C_2 C_2 V_2 O_6 + S_2 S_2 O_2 V_6) \tau_{30} + (C_2 S_2 O_2 O_6 + S_2 C_2 V_2 V_6) \tau_{31} + (S_2 C_2 O_2 O_6 + C_2 S_2 V_2 V_6) \tau_{32} + (S_2 S_2 V_2 O_6 + C_2 C_2 O_2 V_6) \tau_{33} \\
\chi_{53} &= (C_2 C_2 O_2 O_6 + S_2 S_2 V_2 V_6) \tau_{30} + (C_2 S_2 V_2 O_6 + S_2 C_2 V_2 V_6) \tau_{31} + (S_2 C_2 V_2 O_6 + C_2 S_2 O_2 V_6) \tau_{32} + (S_2 S_2 O_2 O_6 + C_2 C_2 V_2 V_6) \tau_{33} \\
\chi_{54} &= (C_2 S_2 V_2 O_6 + S_2 C_2 O_2 V_6) \tau_{30} + (C_2 C_2 O_2 O_6 + S_2 S_2 V_2 V_6) \tau_{31} + (S_2 S_2 O_2 O_6 + C_2 C_2 V_2 V_6) \tau_{32} + (S_2 C_2 V_2 O_6 + C_2 S_2 O_2 V_6) \tau_{33} \\
\chi_{55} &= (S_2 C_2 V_2 O_6 + C_2 S_2 O_2 V_6) \tau_{30} + (S_2 S_2 O_2 O_6 + C_2 C_2 V_2 V_6) \tau_{31} + (C_2 C_2 O_2 O_6 + S_2 S_2 V_2 V_6) \tau_{32} + (C_2 S_2 V_2 O_6 + S_2 C_2 O_2 V_6) \tau_{33} \\
\chi_{56} &= (S_2 S_2 O_2 O_6 + C_2 C_2 V_2 V_6) \tau_{30} + (S_2 C_2 V_2 O_6 + C_2 S_2 O_2 V_6) \tau_{31} + (C_2 S_2 V_2 O_6 + S_2 C_2 O_2 V_6) \tau_{32} + (C_2 C_2 O_2 O_6 + S_2 S_2 V_2 V_6) \tau_{33} \\
\chi_{57} &= (O_2 O_2 C_2 C_6 + V_2 V_2 S_2 S_6) \tau_{30} + (O_2 V_2 S_2 C_6 + V_2 O_2 C_2 S_6) \tau_{31} + (V_2 O_2 S_2 C_6 + O_2 V_2 C_2 S_6) \tau_{32} + (V_2 V_2 C_2 C_6 + O_2 O_2 S_2 S_6) \tau_{33} \\
\chi_{58} &= (O_2 V_2 S_2 C_6 + V_2 O_2 C_2 S_6) \tau_{30} + (O_2 O_2 C_2 C_6 + V_2 V_2 S_2 S_6) \tau_{31} + (V_2 V_2 C_2 C_6 + O_2 O_2 S_2 S_6) \tau_{32} + (V_2 O_2 S_2 C_6 + O_2 V_2 C_2 S_6) \tau_{33} \\
\chi_{59} &= (V_2 O_2 S_2 C_6 + O_2 V_2 C_2 S_6) \tau_{30} + (V_2 V_2 C_2 C_6 + O_2 O_2 S_2 S_6) \tau_{31} + (O_2 O_2 C_2 C_6 + V_2 V_2 S_2 S_6) \tau_{32} + (O_2 V_2 S_2 C_6 + V_2 O_2 C_2 S_6) \tau_{33} \\
\chi_{60} &= (V_2 V_2 C_2 C_6 + O_2 O_2 S_2 S_6) \tau_{30} + (V_2 O_2 S_2 C_6 + O_2 V_2 C_2 S_6) \tau_{31} + (O_2 V_2 S_2 C_6 + V_2 O_2 C_2 S_6) \tau_{32} + (O_2 O_2 C_2 C_6 + V_2 V_2 S_2 S_6) \tau_{33} \\
\chi_{61} &= (V_2 V_2 S_2 C_6 + O_2 O_2 C_2 S_6) \tau_{30} + (V_2 O_2 C_2 C_6 + O_2 V_2 S_2 S_6) \tau_{31} + (O_2 V_2 C_2 C_6 + V_2 O_2 S_2 S_6) \tau_{32} + (O_2 O_2 S_2 C_6 + V_2 V_2 C_2 S_6) \tau_{33} \\
\chi_{62} &= (V_2 O_2 C_2 C_6 + O_2 V_2 S_2 S_6) \tau_{30} + (V_2 V_2 S_2 C_6 + O_2 O_2 C_2 S_6) \tau_{31} + (O_2 O_2 S_2 C_6 + V_2 V_2 C_2 S_6) \tau_{32} + (O_2 V_2 C_2 C_6 + V_2 O_2 S_2 S_6) \tau_{33} \\
\chi_{63} &= (O_2 V_2 C_2 C_6 + V_2 O_2 S_2 S_6) \tau_{30} + (O_2 O_2 S_2 C_6 + V_2 V_2 C_2 S_6) \tau_{31} + (V_2 V_2 S_2 C_6 + O_2 O_2 C_2 S_6) \tau_{32} + (V_2 O_2 C_2 C_6 + O_2 V_2 S_2 S_6) \tau_{33} \\
\chi_{64} &= (O_2 O_2 S_2 C_6 + V_2 V_2 C_2 S_6) \tau_{30} + (O_2 V_2 C_2 C_6 + V_2 O_2 S_2 S_6) \tau_{31} + (V_2 O_2 C_2 C_6 + O_2 V_2 S_2 S_6) \tau_{32} + (V_2 V_2 S_2 C_6 + O_2 O_2 C_2 S_6) \tau_{33}
\end{aligned}$$

(B.1)

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